International Journal of Advanced Computer Research (ISSN (print): 2249-7277 ISSN (online): 2277-7970)
Volume-3 Number-1 Issue-8 March-2013

A linear Fuzzy Goal Programming Method for Solving Optimal Power Generation and Dispatch Problem

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Abstract

This paper presents how fuzzy goal programming (FGP) method can be efficiently used modeling and solving power generation and dispatch (PGD) problems in power system operation and planning horizon. In the proposed approach, the objectives of a problem involved with optimal power flow computation are considered fuzzy in nature in an uncertain decision environment.

In the model formulation of the problem, nonlinear in characteristics of objective functions are first converted into their equivalent linear forms by using Taylor Series approximation technique. Then, the defined fuzzy goals are characterized by their membership functions for measuring the degree of achievement of goal levels of the objectives specified in the decision situation.

In the solution process, minsum FGP methodology is addressed to minimize the deviations from the aspired goal levels and thereby to reach a satisfactory decision on the basis of needs and desires of the decision maker (DM) in the decision making context.

The power generation problem of the standard IEEE 6-Generator 30-Bus System is considered to illustrate the potential use of the approach.

Keywords

Fuzzy goal programming, goal programming, membership function, optimal power flow, Taylor Series approximation.

1. Introduction

The thermal power system operation and planning problems are actually optimization problems with various system constraints in the environment of generating power and dispatching to demand centers. The Optimal PGD problem in power system was first studied by Carpentier [1] in the early 1960s. The general mathematical programming model for optimal power generation was introduced by Dommel and Tinney [2] in 1968. A Comprehensive Survey on environmental power dispatch models developed from 1960s to 1970s was first surveyed in [3]. Thereafter, different mathematical programming approaches have been studied [4-11] for efficient management of PGD problems.

Thereafter, different mathematical programming approaches have been studied [4-11] for efficient management of PGD problems.Since the PGD problem is multiobjective in nature, goal programming (GP) approach [12] based on the satisficing philosophy (coined by Simon [13]), one of the prominent tool for multiobjective decision analysis in crisp decision environment, has been used [14] to obtain the goal-oriented solution for economic-emission power dispatch problems. The crazy swarm optimized economic load dispatch for various types of cost functions has been investigated in [15].

Now, in most of the practical decision situations, it is to be observed that various parameters involved with a problem are often inexact in nature. The most prominent approaches for decision analysis in an uncertain environment is stochastic programming (SP) [16, 17]. The SP approaches to PGD problems have been studied [4, 5] in the past. A multiobjective stochastic search technique for economic load dispatch was presented in [18].

In some decision situations, inexactness of decision parameters are not probabilistic in nature, but they are fuzzily described owing to the imprecise in nature of human judgments as well as inherent imprecision in model parameters. To cope with such a situation, fuzzy programming (FP) approach [19, 20] based on Fuzzy Set Theory (FST) [21] has appeared as robust tool for solving decision problems with multiplicity of objectives.
The use of fuzzy set-theoretic approaches to various practical decision systems, viz., traffic and transportation [22], robot selection [23] and industrial

safety engineering [24] have already been well documented in the literature. In the field of power engineering, although fuzzy programming (FP) [20] methods have been applied to some areas of PGD problems [25, 26], the extensive study in this field is yet to be widely circulated in the literature.

In this article, the FGP [27, 28, 29] approach, which is an extension of conventional goal programming (GP) [12] for multiobjective decision making (MODM) in the area of FP, is considered for modeling and solving optimal PGD problems having the characteristics of nonlinear programming problems in an uncertain decision environment. In the model formulation, the nonlinear objectives are transformed into the linear ones by using Taylor series approximation method [30].Then, the individual best and worst decisions regarding optimization of the objectives are taken into account under the crisply defined system constraints towards fuzzy description of them in the decision making context.

Further, in the sequel of model description, algebraic description of membership functions of the fuzzy goals are considered to reach the solution in terms of degree of achievement of the stated fuzzy goals.
In the solution process, achievement of the highest value (unity) of the membership goals defined for the membership functions to the extent possible on the basis of their weights of importance by minimizing the associated under-deviational variables is considered to reach a most satisfactory decision in the decision making environment. A case example of IEEE 6-Generator 30-Bus System is solved to expound the effectiveness of the proposed approach.

2. General FGP Problem Formulation

In a fuzzy decision making environment, instead of crisp description of objectives and constraints, the fuzzy version of them is taken into consideration and that depends on the needs and desires of DM in the decision making situation.
In the present FGP formulation, the fuzzy version of achieving the aspired levels of the objective goals is considered in the decision making horizon.
Now, the description of fuzzy goals is presented in the following Section 2.1.

2.1 Definition of Fuzzy Goal

Let g_k be the imprecisely defined aspiration level of the k-th objective F_k (**X**), (k = 1,2,...., K). Then, the fuzzy goals may appear in one of the following forms:
$F_k(\mathbf{X}) \gtrsim g_k$ and $F_k(\mathbf{X}) \lesssim g_k$,

where **X** (≥ 0) is the vector of decision variables, g_k is the fuzzy aspiration level of the k-th objective $F_k(\mathbf{X})$ (k = 1, 2,..., K), and $\gtrsim$ and $\lesssim$ refer to fuzziness of the aspiration levels and is to be understood as 'essentially greater than' and 'essentially less than', respectively, in the sense of Zimmermann [20].

2.2 Characterization of Membership Function

Let t_k^l and t_k^u be the lower- and upper-tolerance ranges, respectively, for achievement of the aspired level g_k of the k-th fuzzy goal.
Then, the membership function, say $\mu_k(\mathbf{X})$, for the fuzzy goal $F_k(\mathbf{X})$ can be characterized as [28]:

For $\gtrsim$ type of restriction, $\mu_k(\mathbf{X})$ takes the form:

$$\mu_k(\mathbf{X}) = \begin{cases} 1 & \text{if } F_k(\mathbf{X}) \geq g_k, \\ \dfrac{F_k(\mathbf{X}) - (g_k - t_k^l)}{t_k^l} & \text{if } g_k - t_k^l \leq F_k(\mathbf{X}) < g_k, \\ 0 & \text{if } F_k(\mathbf{X}) < g_k - t_k^l, \end{cases} \quad (1)$$

where (g_k - t_k^l) represents the lower-tolerance limit for achievement of the stated fuzzy goal.

Again, for $\lesssim$ type of restriction, $\mu_k(\mathbf{X})$ becomes:

$$\mu_k(\mathbf{X}) = \begin{cases} 1 & \text{if } F_k(\mathbf{X}) \leq g_k, \\ \dfrac{(g_k + t_k^u) - F_k(\mathbf{X})}{t_k^u} & \text{if } g_k < F_k(\mathbf{X}) \leq g_k + t_k^u, \\ 0 & \text{if } F_k(\mathbf{X}) > g_k + t_k^u, \end{cases} \quad (2)$$

where (g_k + t_k^u) represents the upper-tolerance limit for achievement of the stated fuzzy goal.
Now, formulation of the standard FGP model is presented in the Section 2.3.

2.3 FGP Model Formulation

In an FGP model, the membership functions are transformed into membership goals by assigning the highest degree (unity) as the aspiration level and introducing under- and over-deviational variables to each of them. Then, in the goal achievement function, the under-deviational variables are minimized on the basis of importance of achieving the aspired goal levels in the decision making environment.

Now, since multiple goals are involved with the problem and goals often conflict each other for achieving their aspired levels, the *minsum* FGP [27] model, the simplest and most widely used version of FGP methodology, can be presented as follows.

Find $\mathbf{X} = (x_1, x_2, ..., x_N)$ so as to:

Minimize $Z = \sum \mathbf{w}_k^- d_k^- \; ; k = 1, 2, ..., K$

and satisfy

$$\frac{F_k(\mathbf{X}) - (g_k - t_k^l)}{t_k^l} + d_k^- - d_k^+ = 1$$

$$\frac{(g_k + t_k^u) - F_k(\mathbf{X})}{t_k^u} + d_k^- - d_k^+ = 1$$

subject to $A\boldsymbol{X} \begin{pmatrix} \le \\ = \\ \ge \end{pmatrix} b$,

$$\boldsymbol{X} \ge 0, \tag{3}$$

where $\boldsymbol{X}$ is the decision vector of order (N, 1), A is a real matrix, b is a constant vector and Z represents the fuzzy achievement function consisting of the weighted under- deviational variables d_k^-, and where $d_k^-, d_k^+ (\ge 0)$ with $d_k^- . d_k^+ = 0$, $k = 1,2,..., K$ represent the under- and over-deviational variables respectively, associated with the k-th membership goal, and where $\mathbf{w}_k^- (\ge 0)$ denotes the numerical weight of importance of achieving the k-th fuzzy goal relative to others in the decision-making environment and they are determined as [27]:

$$w_k^- = \begin{cases} \dfrac{1}{t_k^l}, & \text{for the defined } \mu_k \text{ in (1)} \\ \dfrac{1}{t_k^u}, & \text{for the defined } \mu_k \text{ in (2)} \end{cases} \tag{4}$$

Now, in the context of solving the present PGD problem, it may be mentioned that the objectives and some of the constraints are non-linear in nature. Therefore, computational complexity generally arises owing to involvement of non-linear functions. To overcome the difficulty, different linearization approaches have been studied in the area of nonlinear programming [30]. In the present decision situation, since simple quadratic and exponential functions are involved there, the Taylor series approximation method [31] can simply be used for linearization of the nonlinear functions.

2.4 Taylor Series Approximation Method

The Taylor series approximation of a nonlinear function *F(X)*, say, can be presented in the following simple steps [32]:

- **Step 1.** Determine $\mathbf{X}^* = (x_1^*, x_2^*, ..., x_N^*)$, where $\mathbf{X}^*$ indicates the initial solution as an approximate one, around which linear approximation of *F(X)* is taken into account.
- **Step 2.** Transform the function *F(X)* by using first order Taylor series expansion as:

$$F^*(\mathbf{X}) = F(\mathbf{X}^*) + \sum_{n=1}^{N} (x_n - x_n^*) \frac{\partial F(\mathbf{X}^*)}{\partial x_n} \tag{5}$$

Here, in the expression (5), $F^*(\mathbf{X})$ represents the linear approximation of the function *F(X)*.

Now, formulation of PGD problem in the framework of the proposed FGP model is described in the following Section 3.

3. Model Formulation for PGD Problem

The different types of parameters and decision variables involved with PGD problem having generators G_i $(i = 1,2,...,N)$ in a power generation system are introduced as follows:

- **Definition of parameters:**

PD **:** Total power demand (in power-unit (p.u.))

TL **:** Total transmission loss (in p.u) of power in the system

- **Decision variable:**

P_{Gi} **:** Generation of power from the i-th generator G_i

Then, a general PGD problem is defined as follows.

3.1 PGD problem Formulation

A general PGD problem involves two types of objective functions for minimization them subject to a set of system constraints in the decision making environment. The objectives and system constraints are described as follows.

3.1.1 Definitions of Objective Functions

- ***Fuel-Cost Function***

The total fuel- cost ($/hr) function including valve point loading for an PGD problem [11] is defined as follows:

$$TC = \sum_{i=1}^{N}[(\alpha_i P_{Gi}^2 + \beta_i P_{Gi} + \gamma_i) + \left|\delta_i Sin\,[\lambda_i (P_{G_i}^{\min} - P_{G_i})]\right|\,], \quad (6)$$

where *TC* indicates total fuel cost (in $/hr) involved with all the generators and $\alpha_i, \beta_i, \gamma_i$ are fuel cost coefficients associated with the generator G_i , and δ_i and λ_i are the fuel cost coefficients which denote the valve-point effect of the *i*-th generating unit, and where '*min'* stands for minimum.

- **Emission Function:**

The major atmospheric pollutants created by fossil-fuelled thermal units are the sulphur oxides (SO_x), carbon oxides (CO_x) and the oxides of nitrogen (NO_x).

The total emission can be presented as [11]:

$$TE = \sum_{i=1}^{N} 10^{-2}[(a_i P_{Gi}^2 + b_i P_{Gi} + c_i) + d_i e^{h_i P_{Gi}}\,] \quad (7)$$

where *TE* is the total emission (in ton/hr) of all the generators and a_i, b_i, c_i, d_i, h_i are the emission coefficients associated with generation of power from generator G_i.

3.1.2 Description of System Constraints

The system constraints associated with generation of power are defined as follows:

- **Power Balance Constraint:**

The total power generation must have to cover the total demand *PD* and total transmission loss *TL*.

Therefore, the power balance constraint can be expressed as:

$$\sum_{i=1}^{N} P_{Gi} = TL + PD \quad (8)$$

The expression of *TL* can be modelled as a function of generator output, and that can be expressed as:

$$TL = \sum_{i=1}^{N}\sum_{j=1}^{N} P_{Gi} B_{ij} P_{Gj} + \sum_{i=1}^{N} B_{0i} P_{Gi} + B_{00} \quad (9)$$

where B_{ij}, B_{0i} and B_{00} are called *Kron's loss coefficients* or *B-coefficients* [9] associated with the transmission network of a system.

- ***Generation Capacity Constraint*:**

Following the conventional power generation and dispatch system, the constraints on generator outputs can be introduced as:

$$P_{G_i}^{\min} \le P_{G_i} \le P_{G_i}^{\max}\ ,\ i = 1, 2, ..., N \quad (10)$$

where '*min*' and '*max*' stand for minimum and maximum.

Now, it is to be followed that the objectives in (6) and (7) and constraint in (9) are non-linear in nature. Here, linearization technique defined in Section 2.4 can be used to formulate the linear FGP model of the problem. Then, following the proposed procedure, the membership goals of the defined membership functions associated with the linear fuzzy goals and there by modeling of the problem are presented via a case example presented in the following Section 4.

4. A Demonstrative Case Example

The standard IEEE 6-Generator 30-Bus test system [4] is considered for modelling and solving the PGD problem within the framework of the proposed *minsum* FGP approach.

The diagrammatic representation of the single-line diagram of IEEE 6-Generator 30-Bus test system is presented in the Figure 1.

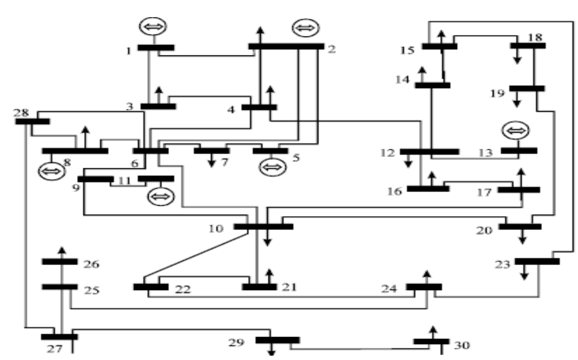

Figure 1: Single-line diagram of IEEE 30-Bus test system

The system in Fig. 1 shows that there are 6 generators and 41 lines. The total system demand for the 30 buses is 2.834 in power unit (p.u).
The data of the parameters associated with the problem are presented in Table 1 – Table 3.

Table 1: Data description for Cost coefficients

Generator \ Coefficient	α	β	γ	δ	λ
G_1	100	200	10	15	6.283
G_2	120	150	10	10	8.976
G_3	40	180	20	10	14.784
G_4	60	100	10	5	20.944
G_5	40	180	20	5	25.133
G_6	100	150	10	5	18.48

Table 2: Data description for Emission coefficients

Generator \ Coefficient	a	b	c	d	h
G_1	6.490	-5.554	4.091	2.0E-4	2.857
G_2	5.638	-6.047	2.543	5.0E-4	3.333
G_3	4.586	-5.094	4.258	1.0E-6	8.000
G_4	3.380	-3.550	5.326	2.0E-3	2.000
G_5	4.586	-5.094	4.258	1.0E-6	8.000
G_6	5.151	-5.555	6.131	1.0E-5	6.667

Table 3: Power generation limits

Limits \ Generator	G_1	G_2	G_3	G_4	G_5	G_6
$P_{G_i}^{\min}$	0.05	0.05	0.05	0.05	0.05	0.05
$P_{G_i}^{\max}$	0.50	0.60	1.00	1.20	1.00	0.60

The *B-coefficients* [9] for the determination of total transmission loss are presented as follows:

$$[B]_{(6,6)} = \begin{bmatrix} 0.1382 & -0.0299 & 0.0044 & -0.0022 & -0.0010 & -0.0008 \\ -0.0299 & 0.0487 & -0.0025 & 0.0004 & 0.0016 & 0.0041 \\ 0.0044 & -0.0025 & 0.0182 & -0.0070 & -0.0066 & -0.0066 \\ -0.0022 & 0.0004 & -0.0070 & 0.0137 & 0.0050 & 0.0033 \\ -0.0010 & 0.0016 & -0.0066 & 0.0050 & 0.0109 & 0.0005 \\ -0.0008 & 0.0041 & -0.0066 & 0.0033 & 0.0005 & 0.0244 \end{bmatrix},$$

$$[B_0]_{(1,6)} = [-0.0107 \quad 0.0060 \quad -0.0017 \quad 0.0009 \quad 0.0002 \quad 0.0030],$$

$$B_{00} = 9.8573E-4$$

(11)

Now, using the data tables, Table 1 and Table 2, and following the expressions in (6) and (7), the objective functions of the problem are obtained as follows:

- ***Fuel-Cost Function***

$$FC = 100P_{G1}^2 + 120P_{G2}^2 + 40P_{G3}^2 + 60P_{G4}^2 + 40P_{G5}^2 + 100P_{G6}^2 + 200P_{G_1} + 150P_{G_2} + 180P_{G_3} + 100P_{G_4} + 180P_{G_5} + 150P_{G_6} + 116.796 \quad (12)$$

- **Emission Function**

$$TE = 10^{-2}(6.49P_{G_1}^2 + 5.638P_{G2}^2 + 4.586P_{G3}^2 + 3.38P_{G4}^2 + 4.586P_{G5}^2 + 5.151P_{G6}^2 + 26.607) - 10^{-2}(5.554P_{G_1} + 6.047P_{G_2} + 5.094P_{G_3} + 3.550P_{G_4} + 5.094P_{G_5} + 5.555P_{G_6}) + (2X10^{-4}e^{2.857P_{G_1}} + 5X10^{-4}e^{3.333P_{G_2}} + 10^{-5}e^{8.00P_{G_3}} + 2X10^{-4}e^{2.00P_{G_4}} + 10^{-6}e^{8.00P_{G_5}} + 2X10^{-4}e^{6.667P_{G_6}}) \quad (13)$$

Then, using *B-coefficients* in (11) and the data Table 3, the power balance constraint, generator output constraints and security constraints are obtained as follows:

- **Power balance constraint**

Using the power demand data, the power balance constraint takes the form:

$$\sum_{k=1}^{N} p_k - TL = 2.834 \quad (14)$$

where TL are obtained as [15]:

$$TL = 0.1382P_{G_1}^2 + 0.0487P_{G_2}^2 + 0.0182P_{G_3}^2 + 0.0137P_{G_4}^2 + 0.0109P_{G_5}^2 + 0.0244P_{G_6}^2 - 0.0598P_{G_1}P_{G_2} + 0.0088P_{G_1}P_{G_3} - 0.0044P_{G_1}P_{G_4} - 0.0020P_{G_1}P_{G_5} - 0.0016P_{G_1}P_{G_6} - 0.0050P_{G_2}P_{G_3} + 0.0008P_{G_2}P_{G_4} + 0.0032P_{G_2}P_{G_5} + 0.0082P_{G_2}P_{G_6} - 0.140P_{G_3}P_{G_4} - 0.0132P_{G_3}P_{G_5} - 0.0132P_{G_3}P_{G_6} + 0.010P_{G_4}P_{G_5} + 0.0066P_{G_4}P_{G_6} + 0.0010P_{G_5}P_{G_6} - 0.0107P_{G_1} + 0.0060P_{G_2} - 0.0017P_{G_3} + 0.0009P_{G_4} + 0.0002P_{G_5} + 0.0030P_{G_6} + 9.8573X10^{-4} \quad (15)$$

- ***Generator output constraints:***

$0.05 \le P_{G_1} \le 0.50$, $\quad 0.05 \le P_{G_2} \le 0.60$,
$0.05 \le P_{G_3} \le 1.00$, $\quad 0.05 \le P_{G_4} \le 1.20$,

$$0.05 \le P_{G_5} \le 1.00, \qquad 0.05 \le P_{G_6} \le 0.60. \tag{16}$$

Now, the individual best solutions of the nonlinear objectives in (12) and (13) and the constraint in (15) by considering it as an objective function are successively found as:

$(P_{G_1}, P_{G_2}, P_{G_3}, P_{G_4}, P_{G_5}, P_{G_6}; FC)$
= (0.121, 0.286, 0.584, 0.993, 0.524, 0.352; 642.794),

$(P_{G_1}, P_{G_2}, P_{G_3}, P_{G_4}, P_{G_5}, P_{G_6}; TE)$
= (0.500, 0.221, 0.758, 0.050, 0.759, 0.590; 0.1916),

and $(P_{G_1}, P_{G_2}, P_{G_3}, P_{G_4}, P_{G_5}, P_{G_6}; TL)$
= (0.050, 0.050, 1.000, 1.200, 0.508, 0.050; 0.0239), respectively.

Now, considering the above power generation decisions as the approximate solutions of the respective nonlinear functions and using the expression in (5), the linear equivalent of the nonlinear expressions in (12), (13) and (15) are successively obtained as:

$$FC^* = 300P_{G_1} + 162P_{G_2} + 260P_{G_3} + 106P_{G_4} + 260P_{G_5} + 206P_{G_6} - 216.79, \tag{17}$$

$$TE^* = 0.4881 - (0.0618P_{G_1} + 0.11106P_{G_2} + 0.0406P_{G_3} + 0.8956P_{G_4} + 0.5552P_{G_5} + 0.06059p_6), \tag{18}$$

and

$$TL^* = 0.00255P_{G_1} + 0.00588P_{G_2} + 0.0107P_{G_3} + 0.02503P_{G_4} + 0.01051P_{G_5} + 0.001P_{G_6} - 0.02267. \tag{19}$$

Then, in the sequel of linear transformation, the power balance constraint in (14) takes the form:

$$0.99745P_{G_1} + 0.99412P_{G_2} + 0.98930P_{G_3} + 0.97497P_{G_4} + 0.98949P_{G_5} + 0.9990P_{G_6} = 2.76543 \tag{20}$$

Now, following the procedure, the aspiration levels and tolerance limits of the defined linear fuzzy goals are computed and presented in the Table 4.

Table 4: Description of Aspiration Levels and Tolerance Limits of Fuzzy goals

Goal	Aspiration Level	Upper Tolerance Limit
Fuel-cost ($/hr)	606.030	646.355
Emission (ton/hr)	0.19418	0.22635

Then, following the procedure, the executable *minsum* FGP model of the problem can be obtained as:

Find $(P_{G_1}, P_{G_2}, P_{G_3}, P_{G_4}, P_{G_5}, P_{G_6})$ so as to:

Minimize $Z = \frac{1}{40.325} d_1^- + \frac{1}{0.0325} d_2^-$

and satisfy

$\mu_1 : (1/40.325)(646.355 - FC^*) + d_1^- - d_1^+ = 1$

$\mu_2 : (1/0.0322)(0.22635 - TE^*) + d_2^- - d_2^+ = 1$

subject to the system constraints defined in (16) and (20),

(21)

where $d_k^-, d_k^+ (\ge 0)$ with $d_k^- . d_k^+ = 0$, $(k=1,2)$ are the under- and over-deviational variables associated with the respective fuzzy goals,.

The LINGO (ver. 12.0) solver (the permissible size of instance is 500 variables and 250 constraints) is used to solve the problem. The model (variable size 10, constraint size 16) is executed in Pentium IV CPU with 2.66 GHz Clock-pulse and 2GB RAM. The required CPU time is 0.01 second.

The resulting decision is presented in the Table 5.

Note: The solution achievement for the use of *additive* FGP approach [29], where maximization of $\sum_{k=1}^{2} \mu_k$, subject to taking $\mu_k \le 1$, without conversion to membership goals, is considered in the same decision environment. Further, without linearization of the nonlinear functions, the goal achievement under the framework of proposed model is also considered to show the effectiveness of the proposed method.

The solutions obtained under the above two approaches are presented in the Table 5.

Table 5: Solution under the Proposed Approach and other Approaches

	Approach / Solution	Proposed	Additive FGP	FGP without Linearization
Generator Output (p.u.)	**P_{G1}**	0.500000	0.050000	0.278962
	P_{G2}	0.5830288	0.050000	0.280951
	P_{G3}	0.3447798	0.280657	0.547752
	P_{G4}	0.2609836	1.200000	0.915079

	P_{G5}	0.5529588	1.000000	0.497716
	P_{G6}	0.5697988	0.228000	0.340786
Total Generation Cost ($/hr)		636.74	654.381	650.711
Total Emission (ton/hr)		0.19442	0.2772	0.23903

The schematic presentation of the results in Table 5 is presented in the Figure 2 and Figure 3.

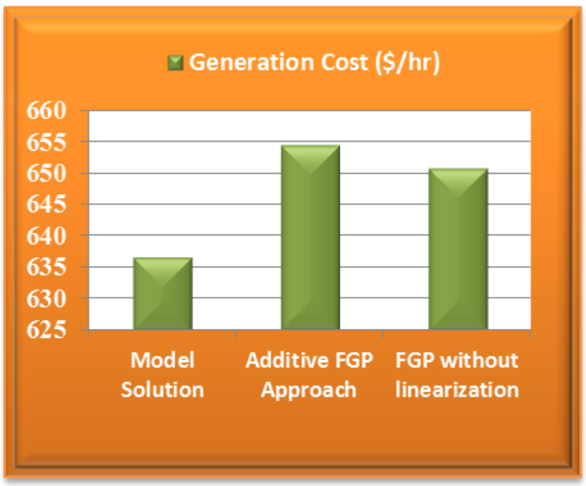

Figure 2: Schematic Presentation of Total Generation Cost under Different Approaches

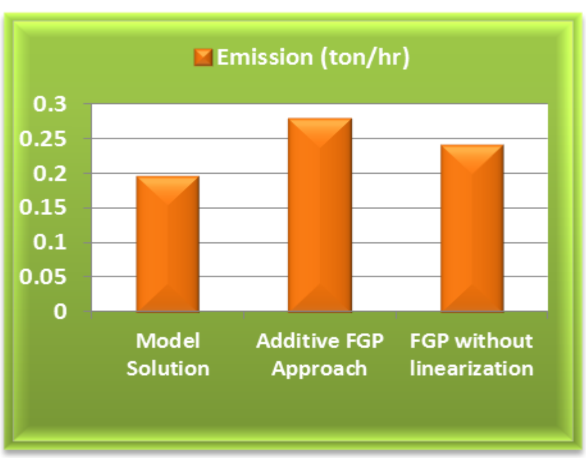

Figure 3: Schematic Presentation of Total Emission under Different Approaches

The graphs show that the both the fuel-cost and emission discharge are minimum under the proposed approach in comparison to the results obtained by using the other two approaches. Therefore, it may be said that the FGP approach presented here is superior over the other ones from the view point of making proper power generation decision with regard to balancing the objectives of minimizing both the power generation cost and environmental emission in an uncertain decision environment.

5. Conclusion

The main advantage of the proposed approach is that the decision trouble with nonlinearity in objectives can easily be avoided here with the use of the Taylor series approximation technique. In the framework of the proposed model, consideration of other objectives and environmental constraints may be taken into account in the context of power plant operations, which may be a problem in future, study. Finally, it is hoped that the solution approach presented here may lead to future research towards proper planning for optimal thermal power generation as well as controlling pollutions for preserving health of the Earth's environment.

Acknowledgement

The authors are thankful to the anonymous Referees for their helpful suggestions to improve the quality of presentation of the paper.

References

[1] J. Carpentier, "Contribution a. '1' etude de Dispatching Economique," Bulletin de la Societe Francaise des Electriciens, 3: 337-431, 1962.

[2] H. W. Dommel and W. F. Tinney, "Optimal Power Flow Solutions," IEEE Transactions on Power Apparatus and Systems, PAS-87(10): 1866-1876, 1968.

[3] H. H. Happ, "Optimal Power Dispatch - A Comprehensive Survey," IEEE Transactions on Power Apparatus and Systems, PAS-96(3): 841-854, 1977.

[4] R. Yokoyama, S. H. Bae, T. Morita., H. Sasaki, "Multiobjective generation dispatch based on probability security criteria", IEEE Transactions on Power Systems,3: 317–324, 1988.

[5] B. H. Chowdhury, H. Rahman, "A Review of Recent Advances in Economic Dispatch", IEEE Transactions on Power Systems, 5: 1248-1258, 1990.

[6] J. S. Dhillon, S. C. Parti, D. P. Kothari, "Stochastic economic emission load dispatch", Electric Power Systems Research, 26: 179–186, 1993.

[7] A. Farag, S. Al-Baiyat., T. C. Cheng, "Economic load dispatch multiobjective optimization procedures using linear programming techniques", IEEE Transactions on Power Systems, 10: 731–738, 1995.

[8] D. Gong, Y. Zhang, C. Qi, "Environmental/economic power dispatch using a hybrid multi-objective optimization algorithm", Electrical Power and Energy Systems. 32: 607–614, 2010.

[9] L. F. Wang, C. Singh, "Environmental/economic power dispatch using a fuzzified multi-objective particle swarm optimization algorithm", Electric Power Systems Research, 77: 1654–1664, 2007.

[10] L. S. Coelho, C-S. Lee, "Solving economic load dispatch problems in power systems using chaotic and Gaussian particle swarm optimization approaches", Electric Power and Energy Systems, 30: 297–307, 2008.

[11] M. A. Abido, "Environmental/Economic Power Dispatch Using Multiobjective Evolutionary Algorithms", IEEE Transactions on Power Systems, 18(4): 1529-1537, 2003.

[12] J. P. Ignizio, Goal Programming and Extensions, Lexington, Massachusetts: D. C. Health, 1976.

[13] H. A. Simon, Administrative Behavior. The Free Press: New York (1945, 4th edition 1997).

[14] J. Nanda, D. P. Kothari, "Lingamurthy, K.S.: Economic-emission load dispatch through goal programming techniques", IEEE Transactions on Energy Conversion, 3: 26-32, 1988.

[15] R. Roy, S. P. Ghoshal, "A novel crazy swarm optimized economic load dispatch for various types of cost functions", International Journal of Electrical Power and Energy Systemsl, 30: 242-253, 2008.

[16] M. K. Luhandjula, "Linear programming under randomness and fuzziness", Fuzzy Sets and Systems, 10: 45 – 55, 1983.

[17] S. Vajda, Probabilistic Programming, Academic Press: 1972.

[18] D. B. Dss and C. Patvardhsn, "New Multi-objective Stochastic Search Technique for Economic Load Dispatch," IEE Proc.-Generation Transmission Distribution**,** 145(6): 747-752, 1998.

[19] R. E. Bellman and L. A. Zadeh, "Decision-Making in a Fuzzy Environment", Management Sciences, 17(4): B141 – B164, 1970.

[20] H. –J. Zimmermann, "Fuzzy Programming and Linear Programming with Several Objective Functions", Fuzzy Sets and Systems, vol. 1, pp. 45-55, 1978.

[21] L. A. Zadeh, "Fuzzy sets", Information and Control, 8(3): 338 – 353, 1965.

[22] D. Teodorovic, "Fuzzy sets theory applications in traffic and transportation," European Journal of Operational Research, 74(3): 379–390, 1994.

[23] T-C.Chu and Y.-C. Lin, "A Fuzzy TOPSIS method for robot selection," International Journal of Advanced Manufacturing Technology, 21: 284-290, 2003.

[24] W. Karwowski and A. Mital, "Potential applications of fuzzy sets in industrial safety engineering," Fuzzy Sets and Systems, 19: 105–120, 1986.

[25] K. Tomsovic, "A Fuzzy Linear Programming Approach to the Reactive Power/Voltage Control Problem," IEEE Transactions on Power Systems, 7(1): 287-293, 1992.

[26] V. Miranda and J. T. Saraiva, "Fuzzy Modeling of Power System Optimal Load Flow", IEEE Transactions on Power Systems, 7(2): 843- 849, 1992.

[27] B. B. Pal and B. N. Moitra, "A goal programming procedure for solving problems with multiple fuzzy goals using dynamic programming" European Journal of Operational Research, 144: 480 – 491, 2003.

[28] B. B. Pal, B. N. Moitra, and U. Maulik, "A Goal Programming Procedure for Fuzzy Multiobjective Linear Fractional Programming Problem", Fuzzy Sets and Systems, 139: 395 – 405, 2003.

[29] R.N. Tiwari, S. Dharmar and J.R. Rao, "Fuzzy goal programming – an additive model", Fuzzy Sets and Systems, 24(1): 27-34, 1987.

[30] M.Avriel, Nonlinear Programming – Analysis and Methods, Prentice-Hall: New Jersey (1976).

[31] Z. M. Odibat, N. T. Shawagfeh, "Generalized Taylor's formula", Applied Mathematics and Computation. 186: 286-293, 2007.

[32] B. B. Pal, S. Sen and M. Kumar, "A linear approximation approach to chance constrained multiobjective decision making problems", IEEE Xplore, d.o.i.:978-1-4244-4786-9/09: 70-75. (2009).

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