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Fuzzy Goal Programming Approach to Chance Constrained Multilevel Programming Problems

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Abstract

This paper presents a fuzzy goal programming (FGP) procedure for solving multilevel programming problems (MLPPs) having chance constraints in large hierarchical decision organizations. In the proposed approach, first the chance constraints of a problem are converted into their respective deterministic equivalent in the decision making context. Then, the objective functions of decision makers (DMs) located at different hierarchical levels are converted into fuzzy goals by introducing an imprecise aspiration level to each of them to make decision in an uncertain environment. In the model formulation, the concept of tolerance membership functions in fuzzy sets for measuring the degree of satisfaction of DMs with regard to achieving the aspired levels of fuzzy goals as well as degree of optimality of the decision vectors controlled by upper-level DMs on the basis of their order of hierarchy in the organizational system. In the solution process, minimization of under deviational variables associated with membership goals defined for the membership functions are considered for achieving the highest membership value (unity) of each of the fuzzy goals to the extent possible on the basis of their weights of importance in the decision making horizon. To illustrate the effectiveness of the proposed approach, a numerical example is solved.

Keywords

Bilevel programming, Chance constrained programming, Fuzzy goal programming, Fuzzy programming, Multilevel programming.

1. Introduction

In real-world decision situations, DMs are often faced with the problem of inexact parameter values due to the imprecision in human judgments as well as inherent inexactness of parameters values of problems.

The two types of prominent approaches for solving above problems are stochastic programming (SP) which deals with probabilistically defined data and fuzzy programming (FP) which deals with imprecisely described data in an uncertain decision environment.

The field of study on SP based on the theory of probability, initially introduced by Charnes and Cooper [1] as chance constrained programming (CCP), has been studied [2,3] extensively and applied to various real-life problems [4-11]. Actually, SP deals with the decision situations where some or all of the parameters of optimization problems are defined by stochastic (random / probabilistic) variables rather than deterministic quantities [5]. In recent years, the methods of multiobjective stochastic optimization problems have become increasingly important in searching solutions of practical decision problems like economics [6], water resource management [7], healthcare [8], transportation [9], agriculture [10], energy systems [11], and other real-life problems.

Again, FP approaches based on the theory of fuzzy sets, initially introduced by Zadeh [12], have been studied [13, 14] deeply from the point of view of potential uses to different real-world problems [15, 16] with imprecisely defined data. Now, in practical decision situations, it has been realized that both the probabilistic and fuzzy data are frequently involved in optimization problems, and both the aspects of SP and FP would have to be taken into account for modelling and solving problems and thereby arriving at optimal decisions. But, consideration of both the aspects in a problem creates a great challenge to DMs for developing efficient solution methods in the current decision making horizon.

The constructive modelling aspects on programming problems under randomness and fuzziness were first studied by Luhandjula [17] in 1983. The methodological development of fuzzy stochastic programming (FSP) [18] approaches for solving linear programming (LP) problems has been surveyed by Luhandjula [19] in 2006. The use of FGP approach [20], an extension of conventional goal programming [21, 22] and as a robust tool for solving multiobjective decision analysis, has been studied in the field of SP by Pal et al. [23] in 2009.

In the field of mathematical programming, multilevel programming MLP [24] was developed to solve decentralized planning problems with multiple decision makers in a hierarchical decision organization. An MLPP can be viewed as an extension of bilevel programming problem (BLPP) [25] for solving large and complex organizational planning problems, where two DMs are located hierarchically at two different decision levels and each control separately a decision vector with the interest of optimizing the individual benefit.

In a hierarchical decision situation, although the execution of decision is sequential from an upper-level to a lower-level, the decision for optimizing the objective of an upper-level DM is often affected by the reaction of a lower-level DM due to his / her dissatisfaction with the decision, because the objectives at different levels often conflict each other owing to individual interests of each of DMs to optimize his / her own objective function. In such case, the problem for proper distribution of decision powers to the DMs is often encountered in most of the hierarchical decision situations.

During 1980s, a considerable number of solution approaches for MLPPs as well as BLPPs as a special case have been deeply studied [24-29] by the pioneer researchers in the field from the viewpoint of their potential use to different real-life hierarchical decision problems such as economic problem [30], agricultural planning [24, 26], electric utility [31]. But, the classical approaches developed so far in the past often lead to a paradox that the decision power of a lower-level DM dominates that of a higher-level DM. To overcome this situation, Wen and Hsu [32] introduced an ideal point dependent solution approach. But their method does not always provide a satisfactory decision in a highly conflicting hierarchical decision situation.

Now, in a hierarchical decision making context, although FP approach to BLPPs having chance constraints has been investigated [33] in the past, it is too early to deep study in the area of FSP from the view point of its potential use in real-life problems. Also, the use of FGP method to MLPPs with chance constraints is in general rare in the literature.

In this paper, the FGP formulation of an MLPP [35] in the field of FSP with the characteristics of randomness in both the coefficient matrix and resource vector is considered. In the proposed solution approach, the notion of the using means and variances in CCP is taken into account to convert the defined chance constraints into their equivalent crisp system constraints. In the process of formulating the model of the problem, the individual best and least solutions of the objectives of each of the DMs located at the different hierarchical decision levels are determined first under the crisply defined system constraints for fuzzy description of the objectives as well as the control vector of the upper-level DMs.

In the FGP model formulation, the membership functions defined for the fuzzy goals are transformed into membership goals by assigning the highest membership value (unity) as the aspiration level and introducing under- and over-deviational variables to each of them. In goal achievement function of the model, attainment of the aspired level of each of the membership goals to the extent possible by minimizing the associated under-deviational variables on the basis of weights of importance of achieving the fuzzy goals is taken into account.

The potential use of the proposed approach is illustrated by a numerical example.

2. Formulation of MLPP

Let the vector of variables $\mathbf{X}(x_1, x_2, \ldots, x_n)$ be involved in the multilevel hierarchical decision system, and let F_k and $\mathbf{X}_k$ be the objective function and control vector of the decision variables of the k th level DM, where $k = 1, 2, \ldots, K\,;\ K \le n$,

and $\bigcup_{k=1}^{K} \{\mathbf{X}_k \mid k = 1,2,\ldots,K\} = \mathbf{X}$.

Then, the generic form of an FMLPP in a hierarchical nested decision structure can be presented as:

Find $\mathbf{X}(\mathbf{X}_1,\ \mathbf{X}_2, \cdots, \mathbf{X}_K)$ so as to:

$$\underset{\mathbf{X}_1}{\text{Max}}\ F_1(\mathbf{X}) = \sum_{r=1}^{K} c_{1r}\mathbf{X}_r \qquad \text{(top-level problem)}$$

for given $\mathbf{X}_1;\ \mathbf{X}_2, \cdots, \mathbf{X}_K$ solve

$$\underset{\mathbf{X}_2}{\text{Max}}\ F_2(\mathbf{X}) = \sum_{r=1}^{K} c_{2r}\mathbf{X}_r \qquad \text{(second- level problem)}$$

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for given $\mathbf{X}_1, \mathbf{X}_2, \cdots, \mathbf{X}_{k-1};\ \mathbf{X}_k$ solves

$\underset{\mathbf{X}_K}{\text{Max}}\ F_K(\mathbf{X}) = \sum_{r=1}^{K} c_{Kr}\mathbf{X}_r$ (K-th level problem)

subject to

$$\mathbf{X} \in S = \{\mathbf{X} \in R^n \mid \Pr[\sum_{j=1}^{n} \hat{a}_{ij} x_j \begin{pmatrix} \geq \\ \leq \end{pmatrix} \hat{b}_i] \geq p_i;\ \mathbf{X} \geq 0, \hat{b}_i \in R^m\} \quad (1)$$

where c_{kr} *(k, r = 1,2,…,K)* are coefficient vectors, 'Pr' indicates the probabilistically defined constraints and $\hat{a}_{ij}, \hat{b}_i \forall\, i, j$ are the normally distributed random variables and p_i *(0 < p_i < 1)* is the satisficing probability level defined for the randomness occurs in the *i*-th constraint. Again, it is assumed that the feasible region S (≠ Φ) is bounded.

Then, the conversion to deterministic (crisp) equivalent of the chance constraints in (1) is described in the following Section 2.1.

2.1 Deterministic Equivalent of Chance Constraints

To determine the deterministic equivalent of chance constraints, the means and variances of $\hat{a}_{ij}$ and $\hat{b}_i \forall i, j,$ are to be defined by considering the distribution function of each of the random variables. Here, in the sequel of finding the value of a random variable, let the random variables are normally distributed, and let *f(y)* be the distribution function of the random variable Y, (say). Then, since *f (y)* is a monotonically non-decreasing function, the value of the corresponding variable y can be found as:

$$f^{-}(\varepsilon) = \{\text{Max y} \mid \Pr(Y \leq y) \leq \varepsilon)\},\ 0 < \varepsilon < 1, \quad (2)$$

where *ε* indicates the satisficing level of probability.

Now, since $\hat{a}_{ij}$ and $\hat{b}_i$ are random variables, the conversion of them to deterministic ones can be described as follows.

Let, $$\hat{y}_i = (\sum_{j=1}^{n} \hat{a}_{ij} x_j - \hat{b}_i) \quad (3)$$

Since, $\hat{y}_i$ is linear combination of the normally distributed random variables, it will also be the normal distribution.

Now, the constraints set in (1) with '≥' type restrictions can be expressed as:

$$\Pr\ [\hat{y}_i \geq 0] \geq p_i,\ i = 1,2,\ldots,m_1;\ m_1 \leq m. \quad (4)$$

For '≤' type of restriction, the probabilistic constraint in (1) takes the form:

$$\Pr\ [\hat{y}_i \leq 0] \geq p_i,\ i = m_1+1, m_1+2, \ldots, m. \quad (5)$$

Here, the three following cases may arise:

(i) If $\hat{a}_{ij}$, *(i = 1,2,…,m₁; j= 1,2,…,n)* are only normally distributed random variables, then the deterministic equivalent expression for '≥' type probabilistic constraints take the form [36]:

$$\sum_{j=1}^{n} E(\hat{a}_{ij})x_j + f^{-1}(p_i)\sqrt{\{\text{var}\,(\hat{a}_{ij})x_j^2} \geq b_i,$$

$i=1,2,\ldots,m_1$ (say) (6)

Proceeding in an analogous way, the another set of non- linear constraints corresponding to the chance constraints in (1) with '≤' type restriction can be obtained as

$$\sum_{j=1}^{n} E(\hat{a}_{ij})x_j + f^{-1}(p_i)\sqrt{\{\text{var}\,(\hat{a}_{ij})x_j^2} \leq b_i,$$

$i=(m_1+1),(m_1+2),\ldots,m$ (7)

(ii) If $\hat{b}_i$ *,(i=1,2,…, m₁)* are only random variables, then as in the above case, the deterministic expression appear as:

$$\sum_{j=1}^{n} a_{ij}x_j - [E(\hat{b}_i) + f^{-1}(p_i)\sqrt{\{\text{var}\,(\hat{b}_i)\}}] \geq 0,$$

$i=1,2,\ldots,m_1$ (8)

(iii) If $\hat{a}_{ij}$, *(i = 1,2,…,m₁; j= 1,2,…,n)* and $\hat{b}_i$ *(i=1,2,…,m₁)* are simultaneously normally distributed random variables, then the deterministic equivalent expression for '≥' type probabilistic constraints take the form:

$$E(\hat{y}_i) + f^{-1}(1-p_i)\sqrt{\text{var}\ (\hat{y}_i)} \geq 0,\ i=1,2,\ldots,m_1 \quad (9)$$

The similar cases arise for consideration of '≤ ' type probabilistic constraints.

Now, FGP formulation of the problem is presented in the following Section 3.

3. FGP Problem Formulation

To formulate the FGP problem of the proposed MLPP in an inexact environment, the fuzzy aspiration levels of the objectives F_k *(k =1,2, …, K)* and decision vectors $\mathbf{X}_k$ *(k=1,2,…,K-1)* are to be determined first. Then, the defined fuzzy goals would have to be characterized by their membership functions for measuring the degree of achievement of the aspired levels of the goals specified in the decision making situation.

3.1 Fuzzy Goal Description

Let $(\mathbf{X}_1^{kB}, \mathbf{X}_2^{kB}, \ldots \mathbf{X}_K^{kB}; F_k^B)$ and $(\mathbf{X}_1^{kW}, \mathbf{X}_2^{kW}, \ldots \mathbf{X}_K^{kW}; F_k^W)$ be the independent best and least solutions, respectively, of the k-th level DM, $k=1,2,...,K$, where $F_k^B = \underset{\mathbf{X}\in S}{\text{Max}} F_k(\mathbf{X})$ and $F_k^W = \underset{\mathbf{X}\in S}{\text{Min}} F_k(\mathbf{X})$; $k=1,2,...,K$.

Then the fuzzy objective goals appear as:

$$F_k \gtrsim F_k^B \text{ , } k= 1,2, ..., K. \tag{10}$$

Now, in the fuzzy decision making context, the lower tolerance limit of the k-th level DM can be introduced as F_k^W $(F_k^W < F_k^B)$, $k=1,2,...,K$.

Again in a hierarchical decision situation, since the benefit of a lower-level DM depends on the relaxation of the decision of the higher-level DMs, the fuzzy goals for the control vectors can be defined as

$$\mathbf{X}_k \gtrsim \mathbf{X}_k^{kB} \text{ ,} k = 1,2, ..., K\text{-}1. \tag{11}$$

Here '$\gtrsim$' indicates the fuzzy version of '$\geq$' in the sense of Zimmermann [14].

Now, let the DMs like to make cooperation each other, and relaxation on the decision of each of the upper-level DMs up to a certain level is made for the benefit of a lower level DM.

The lower tolerance limit of the decision $\mathbf{X}_k$ can be determined as:

$$\mathbf{X}_k^k (\mathbf{X}_k^{kW} < \mathbf{X}_k^k < \mathbf{X}_k^{kB}); k = 1,2,\ldots,(K-1).$$

Then, characterization of membership functions of the defined fuzzy goals is presented in the Section 3.2.

3.2 Characterization of Membership Function

The tolerance membership function for the fuzzy goals $F_k \gtrsim F_k^B$ can be expressed as [20]:

$$\mu_{F_k}(F_k(\mathbf{X})) = \begin{cases} 1, & \text{if } F_k(\mathbf{X}) \geq F_k^B \\ \dfrac{F_k(\mathbf{X}) - F_k^W}{F_k^B - F_k^W}, & \text{if } F_k^W < F_k(\mathbf{X}) < F_k^B \\ 0, & \text{if } F_k(\mathbf{X}) \leq F_k^W \end{cases}$$

$$,k=1,2,...,K. \tag{12}$$

Again, the tolerance membership function for the fuzzy goals $\mathbf{X}_k \gtrsim \mathbf{X}_k^{kB}$ can be presented as:

$$\mu_{\mathbf{X}_k}(\mathbf{X}_k) = \begin{cases} 1, & \text{if } \mathbf{X}_k \geq \mathbf{X}_k^{kB} \\ \dfrac{\mathbf{X}_k - \mathbf{X}_k^k}{\mathbf{X}_k^{kB} - \mathbf{X}_k^k}, & \text{if } \mathbf{X}_k^k < \mathbf{X}_k < \mathbf{X}_k^{kB} \\ 0, & \text{if } \mathbf{X}_k \leq \mathbf{X}_k^k \end{cases}$$

$$,k=1,2,...,K\text{-}1. \tag{13}$$

Now, the FGP model formulation of the problem is presented in the following Section 3.3.

3.3 FGP Model Formulation

In FGP model formulation, the defined membership functions in (12) and (13) are to be transformed into flexible goals by assigning the highest membership value (unity) as the aspiration level and introducing under- and over-deviational variables to each of them.

Then, the *minsum* FGP model of the problem can be presented as [20]:

Find $\mathbf{X}$ $(\mathbf{X}_1, \mathbf{X}_2, \cdots, \mathbf{X}_K)$ so as to:

$$\text{Minimize Z} = \sum_{k=1}^{K} \mathbf{W}_k^- d_k^- + \sum_{k=1}^{K-1} \mathbf{w}_k^- \delta_k^-$$

and satisfy

$$\frac{F_k(\mathbf{X}) - F_k^W}{F_k^B - F_k^W} + d_k^- - d_k^+ = 1 \text{; } k=1,2,...,K \tag{14}$$

$$\frac{\boldsymbol{X}_k - \boldsymbol{X}_k^k}{\boldsymbol{X}_k^{kB} - \boldsymbol{X}_k^k} + \delta_k^- - \delta_k^+ = I \text{ ; } k=1,2,...,K\text{-}1 \tag{15}$$

subject to the system constraint sets defined in (6)-(9). (16)

Here, $d_k^-, d_k^+ \geq 0$, k = *1, 2,..., K*, are the under- and over-deviational variables of the k-th objective goals in (14), and $\delta_k^-, \delta_k^+ \geq 0$, $k=1,2,...,K\text{-}1$ are the vectors of under- and over-deviational variables associated with the respective goals in (15) and Z represents the goal achievement function consisting of the weighted under-deviational variables and vectors of weighted under-deviational variables, where the numerical weights $\mathbf{W}_k^-$ *(k=1,2,...,K)*, $\mathbf{w}_k^-$ *(1,2,...,K-1)* *(> 0)*, represent the relative weights of importance of achieving the goals to their aspired levels, and they are determined as [20]:

$\mathbf{W}_k^- = \dfrac{1}{(F_k^B - F_k^W)}$, for the defined goals in (14),

$\mathbf{w}_k^- = \dfrac{1}{(\boldsymbol{X}_k^{kB} - \boldsymbol{X}_k^k)}$, for the defined goals in (15).

The effective use of the proposed approach is illustrated by a numerical example presented in the Section 4.

4. An Illustrative Example

The following chance constrained tri-level programming problem is considered.

Let x_1, x_2 and x_3 be the decision variables under the control of the first-level, second- level and third-level DMs, respectively.

Then, the MLPP is of the form:

Maximize $F_1 = 6x_1 + 2x_2 + 3x_3$

(first-level problem)

and, for given $x_1; x_2, x_3$ solves

Maximize $F_2(x_1, x_2, x_3) = 5x_1 + 6x_2 + 3x_3$

(second-level problem)

and, for given $x_1, x_2; x_3$ solves

Maximize $F_3(x_1, x_2, x_3) = 2x_1 + 3x_2 + 8x_3$

(third-level problem)

subject to

$$\Pr[\hat{a}_{11}x_1 + \hat{a}_{12}x_2 + \hat{a}_{13}x_3 \le 8] \ge 0.95$$

$$\Pr[x_1 + x_2 + x_3 \le \hat{b}_1] \ge 0.05$$

$$\Pr[\hat{a}_{21}x_1 + \hat{a}_{22}x_2 + \hat{a}_{23}x_3 \ge \hat{b}_2] \ge 0.90$$

(17)

where, $\hat{a}_{11}, \hat{a}_{12}, \hat{a}_{13}, \hat{b}_1, \hat{a}_{21}, \hat{a}_{22}, \hat{a}_{23}, \hat{b}_2$ are independent normally distributed random variables.

Now, in the decision situation, let the means and variances of $\hat{a}_{11}, \hat{a}_{12}, \hat{a}_{13}$ and $\hat{b}_1$ are successively given as (1, 5), (3, 16), (9, 4) and (2.5, 2).

Again, the means and variances of $\hat{a}_{21}, \hat{a}_{22}, \hat{a}_{23}$ and $\hat{b}_2$ are successively given as (5, 3), (6, 4), (8, 5.5) and (7, 5).

Then, following the procedure, the deterministic equivalent of the successive constraints in (17) is obtained as:

$$x_1 + 3x_2 + 9x_3 + 1.645\,(25x_1^2 + 16x_2^2 + 4x_3^2)^{\frac{1}{2}} \le 8,$$

$$x_1 + x_2 + x_3 \le 5.174,$$

and $5x_1 + 6x_2 + 8x_3 + 1.28\,(3x_1^2 + 4x_2^2 + 5.5x_3^2 + 5)^{\frac{1}{2}} \ge 8.$

(18)

Now, following the procedure, the individual optimal solutions of the three successive decision levels are obtained as:

$$(x_1^{1B}, x_2^{1B}, x_3^{1B}; F_1^B) = (0.8482,\ 0.0552,\ 0;\ 5.1998),$$

$$(x_1^{2B}, x_2^{2B}, x_3^{2B}; F_2^B) = (0.4625,\ 0.6327,\ 0;\ 6.1087),$$

$$(x_1^{3B}, x_2^{3B}, x_3^{3B}; F_3^B) = (0.0645,\ 0.0765,\ 0.6166;\ 5.2914),$$

respectively.

Then, the fuzzy goals can be obtained as:

$F_1 \gtrsim 5.1998$, $F_2 \gtrsim 6.1087$, $F_3 \gtrsim 5.2914$,

and $x_1 \gtrsim 0.8482$ and $x_2 \gtrsim 0.6327$.

The lower-tolerance limits of the objective goals are determined as:

$$F_1^W = 1.5144; \quad F_2^W = 1.7431; \quad F_3^W = 1.8621.$$

Now, let the first-level and second-level DMs feel that their respective control variables x_1 and x_2 can be relaxed up to 0.5 and 0.3, respectively, for benefit of the lowest level DM, and not beyond of them. So, $x_1^1 = 0.5\,(x_1^{1W} < 0.5 < x_1^{1B})$ and $x_2^2 = 0.3\,(x_2^{2W} < 0.3 < x_2^{2B})$ act as lower-tolerance limits of the decisions x_1 and x_2, respectively.

Following the procedure and using the above numerical values, the membership functions of the defined fuzzy goals can be constructed by using (12) and (13).

Then, the executable FGP model is obtained as:

Find (x_1, x_2, x_3) so as to:

$$\text{Minimize } Z = \frac{1}{3.6844}d_1^- + \frac{1}{4.3656}d_2^- + \frac{1}{3.4293}d_3^- + \frac{1}{0.3482}\delta_1^- + \frac{1}{0.3327}\delta_2^-$$

and satisfy

$$\mu_{F_1} : (1/3.6844)(6x_1 + 2x_2 + 3x_3 - 1.5144) + d_1^- - d_1^+ = 1$$

$$\mu_{F_2} : (1/4.3656)(5x_1 + 6x_2 + 3x_3 - 1.7431) + d_2^- - d_2^+ = 1$$

$$\mu_{F_3} : (1/3.4293)(2x_1 + 3x_2 + 8x_3 - 1.8621) + d_3^- - d_3^+ = 1$$

$$\mu_{x_1} : (1/0.3482)(x_1 - 0.5) + \delta_1^- - \delta_1^+ = 1$$

$$\mu_{x_2} : (1/0.3327)(x_2 - 0.3) + \delta_2^- - \delta_2^+ = 1$$

subject to the system constraints in (18).

(19)

Here, $d_1^-, d_1^+, d_2^-, d_2^+, d_3^-, d_3^+, \delta_1^-, \delta_1^+, \delta_2^-$ and δ_2^+ (≥ 0), represent the under- and over- deviational variables associated with the respective goals of the model in (19).

The LINGO (ver. 12.0) solver (the permissible size of instance is 500 variables and 250 constraints) is used to solve the problem. The model (variable size 19, constraint size 23) is executed in Pentium IV CPU with 2.66 GHz Clock-pulse and 2GB RAM. The required CPU time is 0.01 second.

The resultant decision is obtained as:

$$(x_1, x_2, x_3) = (0.5075,\ 0.5929,\ 0)$$

with $(F_1, F_2, F_3) = (5.1308,\ 5.899, 2.7937)$.

The achieved membership values are

$$\mu_{F_1} = 0.9968,\ \mu_{F_2} = 0.7371,\ \mu_{F_3} = 0.2717,$$

$$\mu_{x_1} = 0.8980 \text{ and } \mu_{x_2} = 0.8804.$$

The result shows that the values of the objective functions as well as the membership values of the associated fuzzy are achieved on the basis of order of hierarchy introduced in the decision making context. Therefore, a satisfactory decision is achieved here from the view point of proper distribution of decision powers to the DMs in the decision making environment.

Note 1: If the *max- min* fuzzy operator [14] is used to solve the problem (17) in the same decision making environment, where without defining membership goals, maximization of λ in an objective function subject to all the defined membership functions 'less than equal to' λ with $0 \le \lambda \le 1$ is considered, then the solution of the problem by using the Software LINGO (version 12.0) is found as:

$$(x_1, x_2, x_3) = (0.3327,\ 0.4116,\ 0.2753)$$

with $(F_1, F_2, F_3) = (3.6453, 4.959, 4.1026)$.

The obtained membership values are

$$\mu_{F_1} = 0.6607,\ \mu_{F_2} = 0.5166,\ \mu_{F_3} = 0.3866,$$

$$\mu_{x_1} = 0.5623 \text{ and } \mu_{x_2} = 0.5604.$$

A diagrammatic presentation of the membership values achieved under the two different approaches is displayed in the Figure 1.

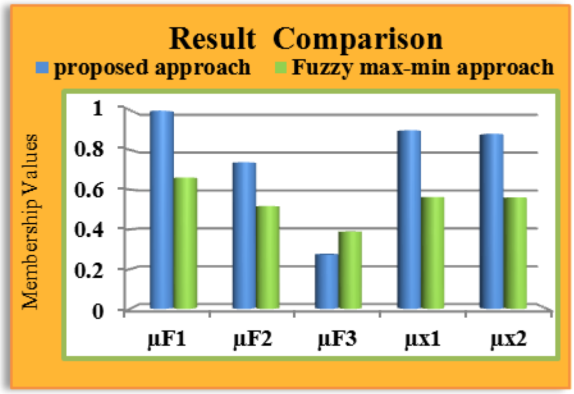

Figure 1: Graphical representation of goal achievement under the two approaches.

The results reflect that, although the hierarchical order of decision powers of the DMs is preserved for the use of *max-min* approach, the solution is inferior in comparison to the solution obtained by using the proposed FGP approach in terms of achieving a better decision of the leader in the decision making environment. Therefore, it may be claimed that the proposed approach is superior over a conventional one to solve problems of hierarchical decision organizations.

5. Conclusion

The main advantage of the proposed approach is that a compromise decision for achievement of aspired goal levels of the individual objectives individually in a hierarchical order can be made on the basis of relative weights of importance by satisfying their admissible tolerance values as defined in the decision making horizon. Further, the proposed FGP model is flexible enough to accommodate other different objectives as defined in the context of making decision, and that depends on the needs and desires of the DMs in an organizational system. Again, consideration of multiplicity of objectives at each decision level in a hierarchical decision system may be taken into account under the framework of the proposed model, which may be a problem in future study.

However, it is expected that the approach presented here can contribute to future study in the field of real-life multiobjective hierarchical decentralized decision problems in uncertain decision environment.

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