

Effect of Blur and Noise on Image Denoising based on PDE

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Abstract

We observe that the effect of noise in image is a challenging issue in the current scenario. Image Denoising has remained a fundamental problem in the field of image processing. Many of the previous research use the basic noise reduction through image blurring. In this paper we survey and analysed different traditional image denoising method using different methods. We also suggest a new approach which provides a heterogeneous way of the above challenging issue. Our approach is the combination of three different approaches first is for blur, second is for noise and finally for blur and noise. After analysing several research works we analyse that not a single method can provide better method for blur and noise both. So our proposed solution can provides betterment in this issue.

Keywords

Image denoising, PDE, SNR, PSNR, Weiner Filter

1. Introduction

Denoising of image data has been a main area of research, which is approaching and being proposed by several researchers using techniques such as wavelets, isotropic and anisotropic diffusion, bilateral filtering, etc. We observe that image contains a large amount of redundancy in plain areas where adjacent picture element have almost the same points and values which means the pixel values are highly correlated [1][2]. In addition, image can contain subjective redundancy, which is determined by properties of a human visual system (HVS). However, HVS present some tolerance to distortion that depending on image contents and viewing condition [2]. Discrete wavelet transform (DWT) offers adaptive spatial-frequency resolution (better spatial resolution at high frequencies and better frequency resolution at low frequencies) that is well suited to the properties of an HVS (Human Visual System) [2]. However, it requires mathematical functions thus; the coding scheme is more complex and not applicable in real-time situation.

All algorithms in the research area of Image denoising is playing an important role in image processing systems. Images mixed with noise are harmful to the progress of image processing. So image denoising is the foundation of other aspects of image processing. There are several algorithms had been proposed recently, such as algorithms based on wavelet transform [3] [4] [5], algorithm based on spatial filters [6] and algorithm based on fuzzy theory [7]. In [8] and [9] the authors used the method of least squares support vector machines and image decomposition respectively. Later, some researchers proposed an algorithm using non-aliasing contourlet transform [10] and partial differential equation [11].

We provide here an overview of Image Compression Technique. The rest of this paper is arranged as follows: Section 2 introduces Image denoising; Section 3 describes about Recent Scenario; section 4 shows the problem domain. Section 5 shows the proposed approach; Section 6 describes Conclusion.

2. PDE

Most physical phenomena, whether in the domain of dynamics, electronics, magnetism, mechanics, image denoising can be described, in general, by formulating partial differential equations (PDEs). It provides the boundary value that may yield solution without computing. In this paper we taken Finite Fourier Transform (FFT) Method for better understanding the PDE.

The FFT method is one of many analytical or numerical techniques in which we find the approximate or analytical solutions to partial differential equations are found by expanding the solution in terms of functions, which is also called basis functions, which determine the coefficients which is unknown in the equation. This application is applying like:

$$\frac{\partial u}{\partial t} = c^2 \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) + E(x, y)$$

reduces the number of spatial variables until only a two-point boundary-value problem or initial-value problem remains, which is solved by standard methods. The FFT method is basically equivalent to the technique of separation of variables; however the FFT method is more flexible and permits a more direct attack on many problems.

The FFT method is applicable to linear boundary-value problems in domains where at least one of the spatial dimensions is finite. Let $\Theta = \Theta(\mathbf{r}, t)$ be the field variable (e.g., temperature or concentration), and let L be a differential operator which contains one or more spatial derivatives and perhaps also a time derivative. Then, a wide variety of partial differential equations can be represented as

$$L\Theta = S(\mathbf{r})$$

where $S(\mathbf{r})$ is a specified function of position. The differential operator is said to be linear if

$$L(a_1\Theta_1 + a_2\Theta_2) = a_1L\Theta_1 + a_2L\Theta_2$$

where a_1 and a_2 are any constants and $\Theta_1(\mathbf{r}, t)$ and $\Theta_2(\mathbf{r}, t)$ are any functions [not necessarily solutions of $L\Theta = S(\mathbf{r})$]. Aside from the requirement of linearity, the other restriction of the FFT method is that the boundaries must correspond to constant values of the coordinates, or coordinate surfaces.

Consider the following PDE

$$\frac{\partial^2 \Theta}{\partial x^2} + \frac{\partial^2 \Theta}{\partial y^2} = 0$$

with the boundary conditions

$$\begin{aligned} \Theta(0, y) &= 0, \\ \Theta(x, 0) &= f_0(x), \end{aligned}$$

The solution can be written in a series expansion as

$$\Theta(x, y) = \sum_{n=1}^{\infty} C_n(y)\Phi_n(x) ,$$

In this equation $\Phi_n(x)$ are given by: $\Phi_n(x) = \sqrt{2} \sin(n\pi x)$, $n = 1, 2, \dots$

The boundary conditions for eigenvalue problems must be of certain type. Thus, the boundary conditions for the equation are usually some combination of

$$\begin{aligned} \Phi &= 0, \\ \Phi' &= 0, \\ \Phi' + A\Phi &= 0, \end{aligned}$$

where primes are used to denote differentiation with respect to x and A is a constant.

3. Literature Review

In 2009, Tongzhou Zhao et al. [12] presented a new approach by using discrete multi-wavelet transform to remote sensing image denoising. The wavelet theories have given rise to the wavelet thresholding method, for extracting a signal from noisy data. According to the authors Multi-wavelets can offer simultaneous orthogonality, symmetry and short support, and these properties make multi-wavelets more suitable for various image processing applications, especially denoising. Denoising of images via thresholding of the multiwavelet coefficients result from pre-processing and the multi-wavelet transform can be carried out by treating the output by the authors. They observe that the Multiwavelet transform technique has a big advantage over the other techniques that it less distorts spectral characteristics of the image denoising. Their experimental results show that multi-wavelet on image denoising schemes outperform wavelet-based method both in subjectively and objectively.

In 2009, Carlos A. J nez-Ferreira et al. [13] observe that the denoising is an important task inside the image processing area. In order to overcome this challenging problem, diverse proposals have been done, like Non-Local means (NL-means) algorithm. Authors present a fast algorithm that uses a preliminary segmentation combined with NL-means for image denoising. Firstly, the algorithm performs a subsampling, called Preliminary Segmentation-Based Subsampling (PSB Subsampling) while reducing the data quantity to be processed, based in the preliminary segmentation information given by the noisy image. This preliminary segmentation finds out an image partition where regions are labeled as significant or non-significant. In a second step, the denoising procedure is done, but NL-means is applied only on some pixels, reducing the data quantity again. The selection of these pixels is done based on information contributed by a segmentation of the subsampled image. According to the authors experimental results show that the implementation of this proposal is quite faster than existing bibliography and it could be used in other image processing tasks like segmentation.

$$(7.1-5b)$$

In 2010, Yan He et al. [14] presents a novel image denoising method based on non-aliasing Contourlet transform (NACT) according to coefficient inter-scale correlation. A noisy image was decomposed into a low frequency approximation sub-image and a series of high frequency detail sub-images at different scale and direction via NACT. In the transform domain, the inter-scale correlation of the signal coefficients was strong, and there was weak inter-scale correlation for noise coefficients, so the noise in the high frequency detail sub-images was removed by using of non-Gaussian bivariate model. According to the authors the result has higher operational efficiency, and it can overcome the aliasing in Contourlet transform and avoid “scratching” phenomenon in the reconstructed image.

In 2010, Xiaotian Wang et al. [15] propose a translation invariant directional lifting (TI-DL) by employing the cycle-spinning based technique to reduce artifacts in denoising results. Moreover, the inefficiency and high computational complexity of the orientation estimation technique in ADL strongly influences the performance. In order to achieve better denoising results, they adopt 2-D Gabor filters for orientation estimation to achieve better orientation estimation results with lower complexity. Experimental results demonstrate that the proposed method achieves state-of-art denoising performance in terms of both objective (PSNR) and subjective (SSIM) evaluation.

In 2010, Guodong Wang et al. [16] propose a denoising method based on adaptive sparse representation in order to avoid estimating the noise variance and remove the white Gaussian noise. It trains the initialized dictionary based on training samples constructed from noised image. The training process is finished by an iteration algorithm which alternates between adaptive sparse representation and dictionary update. Based on the trained dictionary, noise reduction is conducted through adaptive sparse representation of the noised image. Compared with adaptive Wiener filtering and adaptive denoising based on Basis Pursuit, the proposed method could remain more image details and have better performance. With the proposed method, laser electronic speckle interference image could be enhanced and its interference fringe became clearer.

In 2011, Harnani Hassan et al. [17] investigated on suitability wavelet thresholding and translation invariant methods of image denoising to remove

noise using orthogonal wavelet basis. The performance of the image denoising is shown in terms of PSNR and visual performance. The result shown translation invariant gave better PSNR and visual performance than wavelet transform method.

In 2011, Chengdong Wu et al. [18] proposed a novel pavement image denoising method based on shearlet transform. Because that the pavement crack has continuous liner geometrical feature which can be captured by shearlets very efficiently with more directions than wavelets, the pavement image denoising method based on the shearlet transform can obtain a great improvement than traditional method. Background fitting is used to deal with the low frequency component of the image, which can balance the energy distribution of the pavement image. Then coarse scale coefficients of shearlet are selected under multiple thresholds. The coefficients obtained by low threshold is used for reconstruction of the main parts of cracks, and the coefficients obtained by high threshold is employed to extract crack position and direction information, which is fused with the threshold at fine scale to distinguish the noise and fine parts of cracks. The experimental results show that this method can smooth the most of noisy spots but keep the cracks details well and have less pseudo-Gibbs artifacts.

In 2012, Joginder Singh et al. [19] discuss about exist various image de-noising techniques. According to the author orthogonal wavelet is preferred one. However, the orthogonal wavelet transform is not better technique as proper clustering of wavelet coefficients is not possible in this technique. So a better image de-noising technique is needed to have a better SNR and greater image information. So author proposes new work, which is image de-noising by linear minimum mean square-error estimation (LMMSE) scheme is proposed and results show that this method outperforms some of the existing de-noising techniques.

In 2012, Sandeep Dubey et al. [20] discusses about application of denoising process in satellite image data and also in television broadcasting. Image data sets collected by image sensors are generally contaminated by noise. Imperfect instruments, problems with the data acquisition process, and interfering natural phenomena can all degrade the data of interest. Furthermore, noise can be introduced by transmission errors and compression. Thus, denoising is often a necessary and the first step to be

taken before the images data is analyzed. They proposed a novel methodology for image denoising. Image denoising method based on wavelet transform and radial basis neural network and also used concept of soft thresholding. Wavelet transform decomposed image in to different layers, the decomposed layer differentiate by horizontal, vertical and diagonal. For the test of our hybrid method, they used noise image dataset. They consider the data from UCI repository.

In 2012, Meenal Jain et al. [21] proposed three different approaches first is for blur, second is for noise and finally for blur and noise. These approaches are compared by Average absolute difference, signal to noise ratio (SNR), peak signal to noise ratio (PSNR), Image Fidelity and Mean square error. So they can achieve better result on different scenario. They also compare our result on the basis of the above five parameters and the result is better in comparison to the traditional technique.

4. Problem Domain

After studying different approaches we observe that some of the approach provides good images in case of blur, some of the approaches provide food images in case of noise, so there is a need of the approaches which wok better on noise as well as blur.

5. Proposed Approach

If we think of an image then it is in the form double dimension array. The actual intensity is quantized between 0 to 255. Consider the example of figure 5. If we think about the below image it can be quantified in terms of array. The actual images are check with the variations in terms of intensity. In our approach our image is PDE denoised based on three different combinations.



Figure 1: Image

Our three parameters are noise effect, blur and blur with noise effect. If you want to see the difference between the noisy and non-noisy image which is shown in figure 6. It shows the distraction in the real image.



Figure 2: (left) image WITHOUT noise. (Right) image corrupted WITH noise

In the above picture the noise which is added is salt and pepper. If we change the noise type to Gaussian then the effect is different [Figure 7].



Figure 3: (left) image WITHOUT noise. (Right) image corrupted WITH noise

In the above picture the noise which is added is Gaussian. If we change the noise type to Speckle then the effect is different [Figure 8].



Figure 4: (left) image WITHOUT noise. (Right) image corrupted WITH noise

It means we have to adopt a methodology which is better in different noise parameter. In our proposed approach we consider different noise type including salt and pepper, Gaussian and Speckle. So that we can check the noise reduction in the better way.

The process of removing noise from an image is known as noise reduction or denoising. A standard

denoising technique is the convolution of the image with a 2D Gaussian distribution. The formula is shown below:

$$G(x,Y)=1/\sqrt{2\pi\sigma^2} \exp^{-(x^2+y^2)/2\sigma^2}$$

In our approach we also use Average absolute difference, signal to noise ratio (SNR), peak signal to noise ratio (PSNR), image Fidelity and Mean square error for comparing the result. The mean difference is a measure of statistical dispersion equal to the average absolute difference of two independent values drawn from a probability distribution. A related statistic is the relative mean difference, which is the mean difference divided by the arithmetic mean. Signal-to-noise ratio (often abbreviated SNR or S/N) is a measure used in science and engineering that compares the level of a desired signal to the level of background noise. It is defined as the ratio of signal power to the noise power. A ratio higher than 1:1 indicates more signal than noise. The phrase peak Signal-to-Noise Ratio, often abbreviated PSNR, is an engineering term for the ratio between the maximum possible power of a signal and the power of corrupting noise that affects the fidelity of its representation. Because many signals have a very wide dynamic range, PSNR is usually expressed in terms of the logarithmic decibel scale. The PSNR is most commonly used as a measure of quality of reconstruction of lossy compression codecs (e.g., for image compression).

All the parameters taken by us is calculated in the below manner:

First we find the absolute difference.

Step 1: $\text{absdiff} = \text{summation}1/(\text{mdsize}(1)*\text{mdsize}(2));$

Then we find the original mean:

Step 2: $\text{mean} = \text{mean}(\text{limg}(:));$

$\text{tmp} = \text{originalimg} - \text{mean_original};$

$\text{sumsq} = \text{sum}(\text{tmp}(:).^2);$

Then we find the noise:

Step 3: $\text{noise} = \text{restoredimg} - \text{originalimg};$

$\text{mean_noise} = \text{mean}(\text{noise}(:));$

$\text{tmp} = \text{noise} - \text{mean_noise};$

$\text{summation} = \text{sum}(\text{tmp}(:).^2);$

Then we calculate the SNR value:

Step 4: $\text{snr} = 10 * \log_{10}(\text{sumsq} / \text{summation});$

The process is quantizing for the level [0,255] and finds the multiplicative factor also, and then we calculate the PSNR which is shown below:

Step 5: $V1 = \text{tot} * 255;$

$\text{psnr} = V1 / \text{Total};$

$\text{psnr} = 10 * \log_{10}(\text{psnr});$

By this way we can calculate the difference between the original and the noisy image.

MSE is a risk function, corresponding to the expected value of the squared error loss or quadratic loss. MSE measures the average of the squares of the "errors." The error is the amount by which the value implied by the estimator differs from the quantity to be estimated.

We proposed an image denoising method using partial differential equation. In our proposed approach we proposed three different approaches first is for blur, second is for noise and finally for blur and noise. Because in our investigation one methodology is not sufficient to provide better result in all condition.

6. Conclusion

In this paper we present a survey on image denoising. We also concentrate on different denoising techniques and emphasize on the problems. We then observe the problem of noise and blur. We also suggest an efficient solution which is the hybridization to solve the above problem.

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