# Complexity analysis of MIMO sphere decoder using radius choice and increasing radius algorithms 

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#### Abstract

Maximum Likelihood (ML) detection is the optimum method for decoding the received signal vector in communication systems. Its complexity increases with the number of antennas and the constellation size. Sphere Decoding (SD) is an alternate for ML detection. Although SD significantly reduces the complexity of MIMO-ML decoding, its complexity remains too high to apply it for practical systems. As the radius determines the volume of the hyper sphere, choosing a proper radius can be helpful in further reducing the complexity of SD. It provides optimal or suboptimal performance with reduced complexity, as it searches the points which are within the specified radius of the hypersphere. The complexity of the sphere decoder depends on the initial radius selection of the sphere to begin the search process and to update the radius, when no points are found in the specified radius. A look up table of initial search radius is generated using Radius Choice Algorithm. The SD uses this LUT to consider the initial search radius for further processing. The Increasing Radius Algorithm (IRA) is used for updating the radius. The radii of spheres in which expected number of points are some predefined values are obtained. Then using these radii the search begins with IRA. The simulations are performed for constellation size of 4-QAM and 16-QAM with antenna size of 4X4 and 8X8 MIMO. It is shown that the average number of floating point operations are reduced by an amount of $35 \%$ at lower SNR values till 5 dB by reducing the number of nodes visited, without degrading the performance.


## Keywords

MIMO, Sphere Decoding, Radius Choice, increasing radius algorithm.

## 1. Introduction

Because of the huge capacity on a scattering-rich

[^0]wireless channel [1], Multiple-Input Multiple Output (MIMO) system has been extensively researched in the communication community. It is known that ML decoding is the optimum decoding method, but its exponential complexity with the number of transmit antennas and the constellation size. The requirement of exhaustive full search makes it unrealizable in a practical system. Sphere decoding is one of the methods to reduce the complexity of MLD [2] without loss of performance. Although the SD algorithm offers computational efficiency in many communication scenarios [3], [4], it also requires exponential complexity [7] both from a worst-case and from an average point of view. Many algorithms have been proposed to further reduce the complexity of SD.

One of them is tree pruning which sets different radius for different layers, such as increasing radii algorithm (IRA) [7] and probabilistic tree pruning sphere decoding (PTP-SD) [8] Algorithm. The number of visited nodes, which determines the complexity of SD, is reduced by removing the unlikely branches in early stage of sphere search based on the fact that the sphere constraint of the SD algorithm offers a loose necessary condition, especially in the early layers of search.

However, in a large degree, the computational efficiency of those algorithms depends on the choice of initial radius, which determines the search space and influences the complexity of SD significantly. There are many methods to determine a radius for SD. [4] and [6] choose $\infty$ as the initial radius. In this situation, the first point obtained by SD is known as Babai point or zero-forcing decision feedback equalization (ZF-DFE) point. The radius can be updated as the distance between Babai point and the received point. [13] Uses minimum mean square error (MMSE) detection to obtain the initial point. The two methods can ensure that there is at least one point in the sphere, but the radii are often too large due to the poor performance of ZF-DFE and MMSE, which cause the complexity of SD not be significantly reduced. [5] sets the radius as the scaled value of the expected Euclidean norm of noise vector, but how to determine the scaled factor is also a problem. The paper proposes a new algorithm to
determine the radius for SD according to the expected number of points in the sphere. First get the radii of the spheres in which the expected numbers of points are some predefined values and then based on the radii an algorithm proposed.

The MIMO systems have drawn attention because of high spectral efficiency and performance in a given bandwidth. The goal is to minimize BER for the given SNR. Linear detection techniques provide linear complexity but performance is suboptimal. Different methods have been proposed to limit the complexity of the sphere decoding algorithm. Most of them still have a variable complexity depending on the channel conditions. They can be classified in different ways. The first is to modify in the existing algorithm to marginally reduce the complexity associated with additional operations. The second is to simplify the algorithm for specific constellation types. The basic concept is to search N-dimensional hyper sphere of some predefined radius R within the code space.

The recent analysis in [10] has shown that the complexity of sphere decoding algorithm at high SNR for 16-QAM (Quadrature Amplitude Modulation) and 64-QAM modulations can be reasonably implemented with current processors. The minmax approach in [11] utilizes partial information about CSI errors and formulates a worst case, robust decoding scheme. A fast searching algorithm based on List Sphere Decoding (LSD) is proposed in [12]. This algorithm is considered for signals transmitted on multiple antennas. In [13] results show that by applying Phost Enumeration(PE) method with some initial radius is more efficient than by applying Schnor Euchner enumeration.

Well-known soft input soft output (SISO) decoders for MIMO are LSDs. A large stack has to be maintained to calculate the soft outputs. Hence it is most complex and the decoding throughput is less. In [14] a modified K-best Schnor Euchner algorithm is used. In [15] a new technique is introduced which reduces the complexity of sphere decoder substantially. This reduction is accomplished by deconstructing the decoding metric to reduce the number of computations to their minimum and exploiting the structure of lattice representation. The complexity of sphere decoder is measured by number of operations required per visited node multiplied by the number of visited node throughout the search procedure [16]. The complexity can be reduced by either reducing the number of visited node or the
number of operations to be carried out or both of them. A judicial choice of initial radius is made to start the algorithm and this is considered in [17, 18].

The section 2 presents the system model and review the classic SD algorithm. SD algorithm is presented in section 3. The simulation results are provided in section 4 and conclusion is put in section 5

## 2. System model

Consider MIMO systems with M transmit antennas and N receive antennas. The discrete time received signal can be expressed as,

$$
\begin{equation*}
y=\mathrm{H} x+n \tag{1}
\end{equation*}
$$

Where H denotes the channel matrix, $x$ denotes the vector of transmitted symbols, n is the vector of independent and identically distributed noise and $y$ is the vector of received symbols. Sphere decoding is a method for solving the integer least squares problem:

$$
\begin{equation*}
\min \|H x-y\|_{2}^{2} \tag{2}
\end{equation*}
$$

$x \in \mathrm{Z}^{\mathrm{m}}$
Where, $y \in R^{m}$ and $H \in R^{n \times m}$. Note that while $x$ is an integer vector, both the matrix $H$ and vector $y$ are real. As the standard way of solving least squares problems, assuming the matrix $H$ is of full column rank, $H$ is first reduced into an upper triangular matrix using orthogonal transformations, such as the Householder transformation, to obtain the QR decomposition:
$H=Q\left[\begin{array}{c}R \\ 0\end{array}\right]$
Where, $Q \in R^{n \times n}$ is orthogonal and $R \in R^{m \times m}$ is upper triangular. Partitioning $Q=[Q 1$ Q2], where $Q 1$ is $n \times m$ and $Q 2$ is $n \times(n-m)$, we get
$\|H x-y\|_{2}^{2}$
$=\left\|Q\left[\begin{array}{c}R \\ 0\end{array}\right] x-y\right\|_{2}^{2}$
$=\left\|\left[\begin{array}{c}R \\ 0\end{array}\right] x-Q^{T} y\right\|_{2}^{2}$
$=\left\|\left[\begin{array}{c}R \\ 0\end{array}\right] x-\left[\begin{array}{c}Q_{1}^{T} \\ Q_{2}^{T}\end{array}\right] y\right\|_{2}^{2}$
$=\left\|R x-Q_{1}^{T} y\right\|_{2}^{2}+\left\|Q_{2}^{T} y\right\|_{2}^{2}$
Note that the second term in above equation is independent of $x$. Denoting $\hat{y}$ as $Q_{1}^{T} y$, the integer least squares problem is then reduced to following triangular integer least squares problem:

$$
\begin{equation*}
\min _{x \in \mathrm{Z}^{\mathrm{m}}}\|R x-\hat{y}\|_{2}^{2} \tag{6}
\end{equation*}
$$

Sphere decoding solves the above triangular integer least squares problem arising from communication applications. It searches a solution in a predetermined hyper sphere centered at $\hat{y}$.

### 2.1 Sphere Decoder

Sphere decoding, introduced originally by Finke and Pohst in 1985, enumerates all lattice points in a hyper sphere centered at a given vector. It searches a lattice point in a hyper sphere of radius $r$ and centered at $\hat{y}$, that is closest, in Euclidean distance, to the center. Therefore, by restricting the search area, it can reduce the computational complexity of solving the triangular integer least squares problem. Fig. 1 illustrates a hypersphere centered at a vector represented by the hollow point.


Figure 1: Geometric interpretation of hyper sphere in a lattice space.

The sphere radius of $d$ and centered at $y$ can be defined as,
$X=\{x,\|H x-y\| \leq d\}$
condition is equivalent to,
$\|R x-\hat{y}\|_{2}^{2} \leq d^{2}$
Where, $\hat{d}^{2}=d^{2}-\left\|Q_{2}^{T} y\right\|_{2}^{2}$
Since $R$ is upper triangular, so rewriting the above condition, in entry wise as,
$\hat{d}^{2} \geq\left(\sum_{i=1}^{m}\left(\sum_{j=1}^{m} r_{i, j} x_{j}-\widehat{y}_{l}\right)\right)^{2}$
Where $r_{i, j}, j \geq i$ denotes the $(i, j)^{\text {th }}$ entry of $R$. The above equation is expanded to get the equations.
$\hat{d}^{2} \geq\left(\hat{y}_{m}-r_{m, m} x_{m}\right)^{2}+\left(\hat{y}_{m-1}-r_{m-1, m} x_{m}-\right.$
$\left.r_{m-1, m-1} x_{m-1}\right)^{2}+\cdots$
The first term in the right side of above equation depends only on the $\mathrm{m}^{\text {th }}$ entry $x_{m}$ of lattice point $s$, the second term depends on the entries $x_{m}$ and $x_{m-1}$, and so on. By solving, we get,
$\left[\frac{-\hat{d}+\hat{y}_{m}}{r_{m, m}}\right] \leq x_{m} \leq\left[\frac{\hat{d}+\hat{y}_{m}}{r_{m, m}}\right]$
Following the above procedure, obtain the intervals for $x_{m-1}, x_{m-2}$, and so on until $x_{1}$ is reached. . Then it is able to determine all the lattice points in the hyper sphere of radius.

## 3. Sphere decoder algorithm

Inputs: R , where R is upper triangular matrix, $\hat{y}$, where $\hat{y}$ is the y reduced by the QR decomposition. $d$, radius of sphere.
Output: $x$ or null.
Step 1: set $k=m, d_{m}^{2}=d^{2}-\left\|Q_{2}^{T} y\right\|^{2}, \hat{y}_{m \mid m+1}=y_{m}$.
Step 2: (Bounds for $x_{k}$ ), set $U B_{x(k)}=\frac{d_{k}+\hat{y}_{k \mid k+1}}{r_{k, k}}$,

$$
L B_{x(k)}=\frac{-d_{k}+\hat{y}_{k \mid k+1}}{r_{k, k}}
$$

Step 3: (Increase $x_{k}$ ) $x_{k}=x_{k}+1$
If $L B_{x(k)} \leq U B_{x(k)}$, go to step 5; else go to step 4.
Step 4: (Increase $k$ ) $k=k+1$
If $k=m+1$, terminate algorithm, else go to step 3 .
Step 5: (Decrease $k$ ) if $k=1$, go to Step 6; else

$$
\begin{aligned}
& k=k-1, \hat{y}_{k \mid k-1}=\hat{y}_{k}-\sum_{j=k+1}^{m} r_{k, j} x_{j} \\
& d_{k}^{2}=d_{k+1}^{2}-\left(\hat{y}_{k+1 \mid k+2}-r_{k+1, k+1} x_{k+1}\right)^{2}
\end{aligned}
$$

$$
\text { and go to step } 2 \text {. }
$$

Step 6: solution found. Save $x$ and its distance from $y$,

$$
d_{m}^{2}=d_{1}^{2}+\left(y_{1}-r_{1,1} x_{1}\right)^{2} \text { and go to step } 3 .
$$

### 3.1 Increasing Radius Algorithm

Using a schedule of radii and by choosing a smaller radius for the lower dimensions and gradually increasing it, the search space is cut down much earlier than with the sphere decoder. This will reduce the number of points in the search region at the lower dimensions. To reduce the complexity, naturally try to reduce the number of points. However, because of the asymmetry of the region it is possible that the lattice point closest to $x$ does not lie in the search space. For the sphere decoder, the closest point to $x$ inside the hyper sphere is the closest point to x in the entire lattice. For the IRA, however the closest point to x in D is not necessarily the closest point to in the entire lattice. Thus, unlike the sphere decoder, we are not doing ML decoding and are, potentially, incurring a greater BER. Reduced computational complexity is obtained. By the increase in the asymmetry of the search region the computation involved decreased, but simultaneously incur an increased BER.

Algorithm is as follows:
Function DECODE ( $x, H, r$ )
Step 1: $H=Q\left[\begin{array}{c}R \\ 0\end{array}\right]$
Step 2: $t=Q \times x, y=\left[\mathrm{t}_{1}, \ldots \ldots ., \mathrm{t}_{\mathrm{M}}\right]$
Step 3: $D=\Phi, y^{\prime \prime}=r^{\prime}=s=0_{M X I}$
Step 4: While $D=\Phi$
$r=$ GETNEWSCHEDULE
$D=\operatorname{DECREASE}\left(N, y, R, r, y^{\prime \prime}, r^{\prime}, s, D\right)$
Step 5: $\mathrm{s}^{*}=\operatorname{argmin}_{s \in D}\left\|y-H_{s}\right\|^{2}$
Step 6: output s*
function DECREASE $\left(k, y, R, r, y^{\prime \prime}, r^{\prime}, s, D\right)$
Step 1: if $k=0$
$D=D \cup\{s\}$
return
Step 2.elseif $k=N$
$r_{k}^{\prime}=r^{l}, y^{\prime \prime}{ }_{k}=y_{1}$
Step 3.else
$y^{\prime \prime}{ }_{k}=y_{k}-\sum_{j=k+1}^{N} r_{k, j} s_{j}$
$r_{k}^{\prime}=\left(\left(r_{N-k+1}^{\prime}-r^{2}{ }_{N-k}\right)+r^{; 2}{ }_{k+1}\right.$
$\left.-\left(y_{k+1}^{\prime \prime}-R_{k+1, k+1} s_{k+1}\right)^{2}\right)^{1 / 2}$
Step 4: LB $=\max \left(\left[\left(\left(\mathrm{r}^{\prime} \mathrm{k}+\mathrm{y}^{\prime \prime} \mathrm{k}\right) / \mathrm{r}_{\mathrm{k}, \mathrm{k}}\right)-0.5\right]+0.5\right.$, $-0.5 \times(\mathrm{L}-1))$
$\mathrm{UB}=\min \left(\left[\left(\left(\mathrm{r}^{\prime} \mathrm{k}+\mathrm{y}^{\prime \prime} \mathrm{k}\right) / \mathrm{r}_{\mathrm{k}, \mathrm{k}}\right)+0.5\right]-0.5,0.5 \times(\mathrm{L}-1)\right)$
Step 5: for $\mathrm{n}=\mathrm{LB}$ : UB
$\mathrm{s}_{\mathrm{k}}=\mathrm{n}$
D = DECREASE ( $k-1, y, R, r, y^{\prime \prime}, r^{\prime}, s, D$ )
Step 6: return

### 3.2 Radius Choice Algorithm

In Radius Choice algorithm [1], the initial radius can be obtained corresponding to the expected number of points for particular values of SNR.

## Expected Number of Points in a Sphere

The received symbol vector is denoted as $\tilde{x}$ and the actual transmitted symbol vector as $x$. Then, $y-H \tilde{x}=H x+n-H \tilde{x}=H(x-\tilde{x})+n=H_{e}+n$ Where $e=x-\tilde{x}$ is the error symbol vector. Therefore, the components of $y-H \tilde{x}$ are i.i.d. $\bar{N}\left(0, \sigma^{2}+\sigma_{h}^{2}|e|\right)$ random variables and $|y-H \tilde{x}|^{2}$ is a scaled chi-squared distribution with m degrees of freedom, where $\sigma_{h}^{2}$ is the variance of the component of $H$. When the decoding is perfect, $\tilde{x}$ equals to $x$ so $y-H x=n$ is Gaussian random vector whose component is $\bar{N}\left(0, \sigma^{2}\right)$ random variable. With a definite radius $C$ given, obtain the probability that the lattice point $\tilde{x}$ is in the sphere,

$$
\begin{align*}
& F_{\tilde{x}}(C) \\
& =\int_{0}^{\left(C / \sigma^{2}+\sigma_{h}^{2}|e|^{2}\right)}\left(1 / 2^{1 / 2} \Gamma(m / 2)\right)\left(u^{m / 2-1}\right) e^{-u / 2} d u \tag{12}
\end{align*}
$$

$F_{\tilde{x}}(C)=\Phi\left(C /\left(\sigma^{2}+\sigma_{h}^{2}|e|^{2}\right)\right)$

Where, $\sigma^{2}$ is the co-variance and $\sigma_{h}^{2}$ is the covariance of component of channel matrix $H$.
$\sigma^{2}=M\left(L^{2}-1\right) / 6 \rho$
Where, $L^{2}$ is the QAM constellation, $\ulcorner$ (.) is the Gamma function and $\Phi$ (.) is the Cumulative Distributive Function (CDF) of chi-square distribution. A table of initial radius value for any expected number of lattice points for a given value of SNR is formed. A sequence of number of points such as D1, D2, D3 and so on is formed with constant incremental steps. Then the radius values $\mathrm{C} 1, \mathrm{C} 2, \mathrm{C} 3$ and so on respectively are formed using the following equations (12), (13) and (15)for the given value of SNR. For 4-QAM, the equation for the expected number of points is given by,
$D(C)=\sum_{l=0}^{m}\binom{m}{l} F_{\tilde{x}}(C)$
For $16-\mathrm{QAM}$, the equation for the expected number of points is given by,
$D(C)=\left(1 / 2^{m}\right) \sum_{q} \sum_{l=0}^{m}\binom{m}{l} g_{k l}(q) F_{\tilde{x}}(C)$
Where $g_{k l}(q)$ is the co-efficient of $x^{q}$ in the polynomial $\left(1+x+x^{4}+x^{9}\right)^{1}\left(1+2 x+x^{4}\right)^{k-1}$. Similar results can be obtained for 64-QAM and other constellations. The initial radius C 1 is chosen such that it should eliminate the too-large and the toosmall conditions. The too-large condition implies that there are many points within the sphere. Hence, the complexity cannot be reduced effectively. The toosmall condition implies that there is no lattice point within the sphere which leads to repetitive search and hence, increases the complexity. If the search fails with C1, then start the new search with C2 as the initial radius. If there is only one lattice point then the solution will be the ML solution.

## 4. Simulation Results

The simulations are performed for a set of MIMO configurations and two modulation techniques. Fig. 2 is a plot of number of FLOPS vs. SNR for IRA with and without Radius Choice Algorithm. The simulations are carried out for a $4 \times 4 \mathrm{MIMO}$ and for the constellation size of 4-QAM. The figure shows that there is reduction in the number of FLOPS by $35 \%$ at lower SNR region i.e. from zero till 5dB. At higher SNR i.e. above 5 dB the number of FLOPS required remains almost the same.


Figure 2: Plot of average number of Floating point operations versus SNR in dB for 4X4 MIMO and 4-QAM

Fig. 3 is a plot number of FLOPS vs. SNR for IRA with and without Radius Choice Algorithm. The simulations are carried out for an $8 \times 8 \mathrm{MIMO}$ and for the constellation size of 4-QAM. The figure shows that there is reduction in the average number of FLOPS by $39 \%$ at lower SNR region i.e. from zero till 5 dB . At higher SNR i.e. above 5 dB the number of FLOPS required remains almost the same. The simulations are carried for about 50 times and the average is considered for the plotting of the graph.


Figure 3: Plot of average number of Floating point operations versus SNR in dB for 8X8 MIMO and 4-QAM

Table 1 is the look up table for initial radius for the combination of expected number of points and the given value of SNR. These tables are generated using the expression (15) and (16). Here D1 is much less than D2 especially when SNR is high while the difference between two adjacent $D_{i}{ }^{\prime} s$ for $\mathrm{i}>1$ is very small at the entire SNR regime. Here it can be observed that, as the SNR increases the initial radius, from where the search has to be started, decreases. It can also be seen that the initial search radius increases with number of antennas.

Fig. 4 and Fig. 5 are plotted for $8 \times 8$ MIMO configurations for the constellation size of 4 -QAM and 16-QAM. The figures reveal the same performance from zero till 5 dB . At higher SNR i.e. above 5 dB the number of FLOPS required remains almost the same. Fig. 6 is plot of average number of nodes visited versus SNR in dB for $4 \times 4,8 \times 8$ MIMO with 4-QAM, 16-QAM constellations. The graphs revealed that the proposed technique of using the radius choice algorithm for updating the sphere radius has reduced the average number of visited nodes. This in turn has reduced the complexity without degrading the performance.


Figure 4: Plot of average number of Floating point operations versus SNR in dB for 8X8 MIMO and 4-QAM


Figure 5: Plot of average number of Floating point operations versus SNR in dB for 8X8 MIMO and 16-QAM
Table 1: Initial radius Look Up Table for $4 \times 4$ MIMO with 16-QAM when D=1, 2 ...and 8.

| SNR | $\mathbf{D}=\mathbf{1}$ | $\mathbf{D}=\mathbf{2}$ | $\mathbf{D}=\mathbf{3}$ | $\mathbf{D}=\mathbf{4}$ | $\mathbf{D}=\mathbf{5}$ | $\mathbf{D}=\mathbf{6}$ | $\mathbf{D}=\mathbf{7}$ | $\mathbf{D}=\mathbf{8}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 4.038472 | 0.069118 | 0.076905 | 0.082987 | 0.088056 | 0.092442 | 0.096332 | 0.099843 |
| 2 | 2.023659 | 0.063823 | 0.071013 | 0.076630 | 0.081310 | 0.085360 | 0.088952 | 0.092194 |
| 3 | 1.349106 | 0.062058 | 0.069049 | 0.074511 | 0.079062 | 0.082999 | 0.086492 | 0.089645 |
| 4 | 1.009618 | 0.061176 | 0.068067 | 0.073451 | 0.077937 | 0.081819 | 0.085262 | 0.088370 |
| 5 | 0.804156 | 0.060646 | 0.067478 | 0.072815 | 0.077263 | 0.081111 | 0.084524 | 0.087605 |
| 6 | 0.665707 | 0.060293 | 0.067086 | 0.072392 | 0.076813 | 0.080639 | 0.084032 | 0.087095 |
| 7 | 0.565551 | 0.060041 | 0.066805 | 0.072089 | 0.076492 | 0.080302 | 0.083681 | 0.086731 |
| 8 | 0.489329 | 0.059852 | 0.066595 | 0.071862 | 0.076251 | 0.080049 | 0.083417 | 0.086458 |
| 9 | 0.429062 | 0.059705 | 0.066431 | 0.071685 | 0.076063 | 0.079852 | 0.083212 | 0.086245 |
| 10 | 0.379964 | 0.059587 | 0.066300 | 0.071544 | 0.075914 | 0.079695 | 0.083048 | 0.086075 |

## 5. Conclusion and Future Work

Although Sphere Decoding (SD) is proposed as an alternative for ML decoding, its complexity is still too high to apply it into practical systems. Because the radius determines the volume of the hyper sphere, choosing a proper radius can be very helpful in further reducing the complexity of SD. The proposed idea to consider Choice Algorithm has reduced the
complexity. This is achieved in turn by reducing the. The number of nodes visited. The simulations are performed for the initial radius for the search process from the Look Up Table (LUT) generated by using the Radius constellation size of 4-QAM and 16-QAM and antenna size of $4 \times 4$ and $8 \times 8 \mathrm{MIMO}$. It is shown that the number of floating point operations reduced significantly by an amount of $35 \%$ at lower SNR values till 6 dB by reducing then the number of nodes visited without degrading the performance.

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Figure 6: Plot of average number of nodes visited versus SNR in dB for 4X4, 8X8 MIMO and 4-QAM, 16-QAM constellations


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