

Error Code Correction Using Reed Solomon Codes: A Review

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Abstract

In wireless communication systems reducing bit/frame/symbol error rate is critical. If bit error rates are more than in wireless communication system our aim is to reduce the error by applying various encoding methods on the data which is going to be transferred. Various channel coding techniques for error detection and correction helps the communication system designers to minimize the effects of a noisy, attenuated and distorted data transmission channel. In this paper our focus is to study and analysis of the performance of Reed-Solomon code that is used to encode the data stream in digital communication. The performances were evaluated by applying to different phase shift keying (PSK) modulation scheme in Noisy channel. Reed-Solomon codes are one of the best for correcting burst errors and find wide range of applications in digital communications and data storage. Reed-Solomon codes are good coding technique for error correcting, in which redundant information is added to data so that it can be recovered reliably despite errors in transmission or retrieval. The error correction system used is based on a Reed-Solomon code. These codes are also used on satellite and other communications systems.

Keywords

Reed-Solomon (RS), Galois Field (GF), Generator Polynomial $g(x)$, Bit Error Rate (BER).

1. Introduction

REED-SOLOMON (RS) codes [15] are among the most celebrated forward error correcting codes. The RS codes are currently used in a wide variety of applications, start from satellite communications to data storage systems. RS codes have been taken as outer codes in the third-generation (3G) wireless Standard, CDMA2000 high-rate broadcast packet data air interface, and are expected to be used as outer codes in concatenated coding schemes for future fourth-generation (4G) wireless systems. The error correction system used on CD's and DVD's is based on a Reed-Solomon code. RS codes, in which redundant information is added to data so that it can

be used to recover errors in transmission or storage and retrieval. The error correction system used on CD's and DVD's is based on a Reed-Solomon code. A Reed-Solomon code can be defined as an (n, k) code, where n is the total length of the code word and k is the number of information words in complete code word. As shown in figure 1.1.

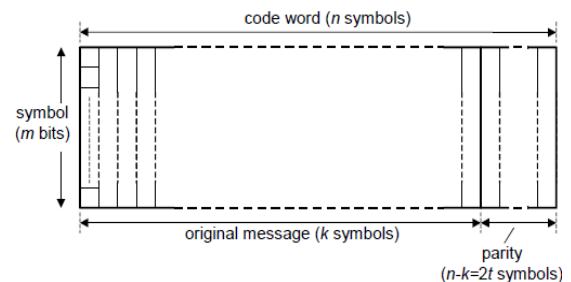


Figure 1.1

Maximum-likelihood (ML) decoding of linear codes and RS codes is NP-hard. It remains an open problem to find polynomial-time decoding algorithms with near ML performance. A soft-decision ML decoding algorithm was proposed by Vardy and Beery. Further modifications of this algorithm were also studied. Guruswami and Sudan (GS) invented a polynomial-time list decoding algorithm for RS codes capable of correcting beyond half the minimum distance of the code. Koetter and Vardy (KV) developed an algebraic soft-decision decoding (ASD) algorithm for RS codes based on a multiplicity assignment scheme for the GS algorithm. Alternative ASD algorithms, such as the Gaussian approximation algorithm by Parvaresh and Vardy and the algorithm by El-Khomy and McEliece based on the Chernoff bound, have better performance. Jiang and Narayanan (JN) developed an iterative algorithm based on belief propagation for soft decoding of RS codes. This algorithm compares favorably with other soft-decision decoding algorithms for RS codes and is a major step toward message passing decoding algorithms for RS codes. In the JN algorithm, belief propagation is run on an adapted parity-check matrix, where the columns in the parity-check matrix corresponding to the least reliable independent bits are reduced to an identity submatrix. The order statistics decoding algorithm by Fossorier and Lin also sorts the received bits with respect to their reliabilities and reduces the columns in the generator matrix corresponding to the most reliable

bits to an identity submatrix. This matrix is then used to generate (permuted) codewords using the most reliable bits. Other soft-decoding algorithms for RS codes include the generalized minimum distance (GMD) decoding algorithm introduced by Forney, the Chase II algorithm, the combined Chase II-GMD algorithm, and successive erasure-error decoding.

2. Literature Review

In [1] authors, The number of monomials required to interpolate a received word in an algebraic list decoder for Reed–Solomon codes depends on the instantaneous channel error, and not only on the decoder design parameters. The implications of this fact are that the decoder should be able to exhibit lower decoding complexity for low-weight errors and, consequently, enjoy a better average-case decoding complexity and a higher decoding throughput. On the basis of analytical side, this paper studies the dependence of interpolation costs on instantaneous errors, in both hard- and soft-decision decoders. On the algorithmic side, it provides an efficient interpolation algorithm, based on the state-of-the-art interpolation algorithm, that enjoys reduced running times for reduced interpolation costs. Which conclude that the results presented in this paper provide a general framework to improve, and quantify the improvement, of the average complexity of RS code algebraic list decoders. However, each of the algorithmic/analytical tools described here can be further studied and extended. The improved interpolation algorithm can be further optimized for time and space complexities. More refined channel models, e.g., additive noise channels over PSK and QAM signal constellations, can be considered for bounding the resulting posterior probabilities and in turn the average interpolation costs. In addition, extending the analytic and algorithmic results from RS codes to algebraic-geometry codes is an interesting yet still open area.

In [2] authors, algebraic soft-decision decoding (ASD) algorithm are a polynomial-time soft decoding algorithm for Reed- Solomon (RS) codes. It outperforms both the algebraic hard decision decoding (AHD) and the conventional unique decoding algorithms; this paper proposes a progressive ASD (PASD) algorithm that enables the conventional ASD algorithm to perform decoding with an adjustable designed factorization output list size (OLS). The OLS is enlarged progressively leading to an incremental computation for the interpolation and an

enhanced error-correction capability. Multiple factorizations are performed in order to find out the intended message polynomial which will be validated by a cyclic redundant check (CRC) code. The incremental interpolation constraints are introduced to characterize the progressive decoding. The validity analysis of the algorithm shows the PASD algorithm is a natural and computationally saving generalization of the ASD algorithm, delivering the same interpolation solution. The average decoding complexity of the algorithm is further theoretically characterized, revealing its dependence on the channel condition. The simulation results further validate the analysis by showing that the average decoding complexity can be converged to the minimal level in a good channel condition. Finally, performance evaluation shows the PASD algorithm preserves the error-correction capability of the ASD algorithm. Which conclude that the progressive algebraic soft decision decoding algorithm for the RS codes. It has been given that the designed factorization OLS can be enlarged progressively, leading to a progressive interpolation for which the decoding computation can be performed incrementally. More than preserving the error-correction capability, it adjusts the decoding complexity according to the quality of the information which is received. The validity analysis of the algorithm showed that the progressive interpolation is a natural generalization of the original interpolation problem. With the same decoding OLS, they deliver the same solution. It is important to acknowledge that such a progressive decoding approach is on the expense of the system memory. The complexity analysis further characterized the channel dependence feature of the proposed algorithm. With improving the channel condition, the algorithm will perform more decoding events with a small OLS value and reduce the average decoding complexity. Simulation results reaffirmed the analysis and revealed that in the practically interested SNR region, significant complexity reduction can be achieved. Finally, performance evaluation showed that the PASD algorithm retains the error-correction performance of the ASD algorithm.

In [15] authors, proposed a new area-efficient truncated inversion less Berlekamp-Massey architecture for the Reed-Solomon (RS) decoder, where RS decoder is one of the forward error correction techniques. The area-efficient feature of the proposed architecture is obtained by truncating redundant processing elements in the key equation solver (KES) block using the BM algorithm. This

increases the hardware utilization of the processing elements used to solve the key equation and reduces the hardware complexity of the KES block. The proposed TiBM architecture has the lowest hardware complexity compared with conventional KES architectures. Which concludes that area-efficient TiBM architecture and evaluated its performance for the RS (255,239) decoder design, which can correct up to eight bit error in one block. The TiBM architecture has very low complexity in comparison with the conventional KES architectures. TiBM architecture is well suited for high-speed low-complexity RS decoder design.

In [3] authors, study of codes for power line communications has garnered much interest over the past decade. Various types of codes such as permutation codes, frequency permutation arrays, and constant composition codes have been proposed over the years. In this paper, he study a type of code called a bounded symbol weight code which was first introduced by Versfeld et.al. in 2005, and a related family of codes that we term constant symbol weight codes. We provide new upper and lower bounds on the size of bounded symbol weight and constant symbol weight codes. He also give direct and recursive constructions of codes for certain parameters. Which concludes that he derive the asymptotic estimates of the sizes of symbol weight codes. We also provide means of obtaining lower bounds on such codes and show that it is possible to provide symbol weight codes with the minimal possible symbol weight via recursive constructions, given he start with a known such code. Finally, we provided constructions of asymptotically good constant symbol weight codes. It remains open to determine families of codes with positive rate and positive relative distance with symbol weights that are optimal or close to the optimal value of $[n/q]$.

In [4] authors, The (n, k) information dispersal algorithm (IDA) is the technique of converting a file into n pieces of shadows, and any k out of the n shadows sufficient for reconstructing the file. The IDA is applicable to the distributed communication and storage systems. The systematic case of (n, k) IDA, the encoder reads k input symbols and then generates m parity symbols in every iteration. In encoding, we observe that several FNT-based algorithms employ n -point FNT within $\Theta(n \log n)$. However, for small m , i.e., the high code rate case, the n -point FNT may re-generate the input symbols in the resulting symbols. To remove those redundant

overhead, the proposed algorithm employs m -point FNTs to reduce the complexity from $\Theta(n \log n)$ to $\Theta(n \log m)$. In decoding, since the decoder only needs calculating the m erased symbols at most, we can also employ m -point FNT to achieve better performance by Fermat algorithm.

In [5] authors, presents a novel efficient algorithm for the estimation of sparse channel impulse response (CIR) is addressed for OFDM systems. The innovation of this algorithm comes from the fact that it equivalently sees the CIR estimation problem as a decoding one. To do so, it exploits first the channel sparsely through the modeling of the sparse CIR as a Bernoulli-Gaussian process. Then, using the relationship between the Reed-Solomon codes and the OFDM modulator it efficiently estimates the sparse CIR using directly the decoding of the OFDM received signal. The obtained simulation results highlight that using the proposed algorithm gives good estimation performance in terms of mean squares error on the sparse CIR estimates. Which concludes that using a multicarrier OFDM transmission system, pilot tones can be seen as virtual RS code words. Author propose to model that sparse CIR as a Bernoulli-Gaussian channel in order to efficiently recover the CIR parameters by an adequate modified version of the RS-PGZ decoding algorithm. The obtained simulation results show the efficiency of our proposed CIR estimation method for sparse channels making it a good competitor for emerging sparse channel estimation algorithms.

In [7] authors, Reed-Solomon (RS) codes are widely used as forward correction codes (FEC) in digital communication and storage systems. Correcting errors of RS codes have been extensively studied in both academia and industry. However, for burst-error correction, the research is still quite limited due to its ultra-high computation complexity. In this brief, starting from a recent theoretical work, a low-complexity reformulated inversion less burst-error correcting (RiBC) algorithm is developed for practical applications. Then, based on the proposed algorithm, a unified VLSI architecture that is capable of correcting burst errors, as well as random errors and erasures, is firstly presented for multi-mode decoding requirements. This new architecture is denoted as unified hybrid decoding (UHD) architecture. It will be shown that, being the first RS decoder owning enhanced burst-error correcting capability, it can achieve significantly improved error correcting capability than traditional hard-decision decoding

(HDD) design. Which concludes that In this brief, a high-speed RiBC algorithm for RS code burst-error correcting, and a UHD architecture that can support three different decoding modes are proposed. Comparison results show that the UHD decoder can achieve enhanced capability of correcting long burst of errors with good hardware efficiency.

In [8], unique word orthogonal frequency division multiplexing (UW-OFDM) inherently introduces a complex number Reed Solomon (RS) code. Originally, the code generator matrix of systematic coded UW-OFDM had been designed rather intuitively by minimizing the mean redundant energy. In this work we justify this approach by applying a cost function that incorporates the overall transceiver chain including a linear minimum mean square error (LMMSE) data estimator. In addition to the LMMSE estimator we investigate a nonlinear sphere detection (SD) receiver for both systematic and nonsystematic coded UW-OFDM. We study and interpret the estimators' performance and their diverse ability to exploit the redundant energy. Which conclude that we compared the LMMSE data estimator and the SD receiver for systematic and nonsystematic complex number RS coded UW-OFDM systems. The characteristics of the two estimators are discussed for the AWGN as well as for frequency selective channels. Furthermore, our original code generator matrix design approach for systematic coded UW-OFDM is justified by the introduction and minimization of a different cost function which focuses on the overall transceiver performance rather than on the redundant energy.

In [9] authors, to reduce the complexity of algebraic soft-decision decoding (ASD) of Reed–Solomon (RS) codes, re-encoding and coordinate transformation can be applied. For an (n, k) code, the re-encoding was implemented as applying erasure decoding to the k most reliable code positions previously. Such re-encoding can occupy a significant part of the overall decoder area. In this brief, we propose to choose the first k positions and implement the re-encoding in the low-complexity Chase (LCC) ASD algorithm by systematic encoding, which can be done by simple constant multipliers. Moreover, novel schemes are developed to modify the following interpolation and code word recovery steps in the case that systematic symbols need to be flipped to form the test vectors in the LCC decoding. Without any performance loss, the proposed schemes can lead to 15.5% higher efficiency in terms of throughput-over-area ratio in the LCC

decoder with eight test vectors for a $(255, 239)$ RS code over GF(28). Which conclude that to use systematic re-encoding in the LCC decoder and developed novel schemes to accommodate the flipping of systematic code positions? Systematic re-encoding is much simpler than erasure re-encoding, and the required modifications on the following decoding steps have small overhead. As a result, the proposed decoder can achieve much higher efficiency than prior designs. Our future work will exploit if systematic re-encoding can be employed in general ASD decoders.

In [16] authors, present an iterative soft-decision decoding algorithm for Reed–Solomon (RS) codes offering both complexity and performance advantages over previously known decoding algorithms. algorithm is a list decoding algorithm which combines two powerful soft-decision decoding techniques which were previously regarded in the literature as competitive, namely, the Koetter–Vardy algebraic soft-decision decoding algorithm and belief-propagation based on adaptive parity-check matrices, recently proposed by Jiang and Narayanan. Building on the Jiang–Narayanan algorithm, he presents a belief-propagation- based algorithm with a significant reduction in computational complexity. he introduce the concept of using a belief-propagation- based decoder to enhance the soft-input information prior to decoding with an algebraic soft-decision decoder. Which concludes that algorithm is based on enhancing the soft reliability channel information before passing them to an algebraic soft-decision decoding algorithm. This was achieved by deploying the Jiang and Narayanan algorithm, which runs belief-propagation on an adapted parity-check matrix. Using the Koetter–Vardy algorithm as the algebraic soft-decision decoding algorithm, algorithm has impressive coding gains over previously known soft-decision decoding algorithms for RS codes. By comparing with averaged bounds on the performance of ML decoding of RS codes, we observe that our algorithm achieves a near optimal performance for relatively short, high-rate codes. He introduced some modifications over the JN algorithm that resulted in better coding gains. He presented a low complexity adaptive belief-propagation algorithm, which results in a significant reduction in the computational complexity. The performance of our algorithm was studied for the cases when the interpolation cost of the algebraic soft-decision decoding algorithm is both finite and infinite. A small loss in coding gain results when using manageable interpolation costs. The coding gain of

the presented algorithm is larger for channels with memory. Algorithm could also be viewed as an interpolation multiplicity assignment algorithm for algebraic- soft decoding.

3. Problem Statement

A simple encoding and decoding algorithm for RS code is presented in this paper is based on the fact that the code word used in Euclid's algorithm is a non-systematic RS code. It uses the recursive extension to compute the remaining unknown syndromes. Finally, the message symbols are thus obtained by only subtracting all known syndromes from the coefficients of the corrupted information polynomial. In other words, a polynomial division used to evaluate the messaging polynomial. The speed of the new Euclidean-algorithm-based decoding approach is shown to be slightly faster than others algorithms [11]. It can also be utilized to find the errata locator polynomial from Berlekamp-Massey algorithm [12] provided that the message vector has the same format as the one given previously.

4. Conclusion

The reed Solomon code has been widely applied obtain gains in coding and diversity. Coding gain and diversity techniques, which are used to reduce the effect of fading. We have given analytical performance results for RS codes over spatially correlated channels. Based on this expression, an analytical estimate for bit error probability and frame error probability is computed. RS codes with orthogonal frequency division multiplexing (OFDM), for high data rate wireless applications OFDM is widely used because of its ability to combat inter symbol interference (ISI). In this work, we used the LMMSE data estimator and the receiver for systematic numbers RS coded OFDM systems. The characteristics of the two estimators are discussed for the AWGN as well as for frequency selective channels. Furthermore, our original code generator matrix design approach for systematic coded OFDM system.

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