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Reduced Order Modelling and Optimal Control of Fluid Flow Instability

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Abstract

This paper presents a framework for developing a Fluid–Sensor–Actuator system for optimal control of fluid instability. The governing Navier – Stokes equations are linearized and reduced to a finite dimensional state space form suitable for developing a optimal control scheme. The aim of the work is to study the unstable modes of the system and stabilize the system using optimal control methods.

Keywords

Feedback Control, Flow Control, Optimal Control, Flow Instability, Proper Orthogonal Decomposition

1. Introduction

Transition from Laminar state to Turbulence remains a fundamental problem in Fluid Dynamics. A lot of effort has gone in understanding the physics and the instability causing the transition from Laminar Flow to Turbulent Flow. In all Fluid Flow systems especially related to Aerospace Engineering, Laminar Flow is preferred over Turbulent Flow due to low drag and savings in cost which would otherwise be high if the flow was Turbulent.

Flow transition is a highly Non – Linear phenomena governed by the Navier–Stokes Equation and still not fully understood. On the other hand, the Linear Stability problem is well researched and understood [1]. Also, it has been shown that finite amplitude disturbances in two dimensional flows grow and sustain instability in three dimensional flows [2]. It is speculated that this 2D/3D coupling may be responsible for flow transition. Therefore, study of the linear two dimensional instability is a good hunting ground for understanding flow transition and control.

If a proper Feedback Control System is designed for controlling the linear instability in the flow, it may render the flow stable for a wide range of Reynolds number and thereby delay or suppress transition.

Previous work done in this area focused mainly on wave cancellation techniques as discussed by Joslin et al. [3]. The basic idea was to study the wave structure of the instability and cancel the primary waves using superposition principle with an exact out of phase wave. The disadvantage of this method is that the parameters determining the instability wave structure should be known or measurable to be able to construct the cancellation wave.

Unlike the wave superposition principle, this paper builds on the Fluid – Sensor – Controller – Actuator system proposed by Joshi et al. [4]. This approach differs from the wave superposition technique and focuses on measuring the system instability and suppressing them via a feedback loop, thereby making the entire system stable.

Developing such a framework gives the control system designer freedom to choose from a variety of control schemes and develop a system which is least complex, robust and requires least amount of control effort.

2. Physical Model

We study the prototype incompressible three dimensional flow between flat plates of dimensions shown in Figure 1.

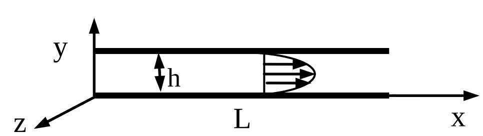

Figure 1: Physical Model

The flow is governed by Linearized Small Perturbation Navier – Stokes equations given as:

$$\left(\frac{\partial}{\partial t} + U\frac{\partial}{\partial x}\right)u + v\frac{\partial U}{\partial x} = -\frac{\partial p}{\partial x} + \frac{1}{Re}\nabla^2 u \qquad (1)$$

$$\left(\frac{\partial}{\partial t} + U\frac{\partial}{\partial x}\right)v = -\frac{\partial p}{\partial y} + \frac{1}{Re}\nabla^2 v \qquad (2)$$

$$\left(\frac{\partial}{\partial t} + U\frac{\partial}{\partial x}\right)w = -\frac{\partial p}{\partial z} + \frac{1}{Re}\nabla^2 w \qquad (3)$$

And the Continuity equation is given as:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \qquad (4)$$

Where u, v and w are the perturbation velocities in the x, y and z-directions respectively. The primary flow velocity profile is denoted by U and the centerline velocity by U_c. The above system of equations can be reduced to:

$$\left(\frac{\partial}{\partial t} + U\frac{\partial}{\partial x}\right)\nabla^2 v - \frac{d^2 U}{dy^2}\frac{\partial v}{\partial x} = \frac{1}{Re}\nabla^2\nabla^2 v \qquad (5)$$

$$\left(\frac{\partial}{\partial t} + U\frac{\partial}{\partial x}\right)\omega_y - \frac{1}{Re}\nabla^2\omega_y = -\frac{dU}{dy}\frac{\partial v}{\partial z} \qquad (6)$$

No slip boundary conditions are applied at the lower and the upper wall at $y = 0$ and $y = h$. Also, blowing and suction is enforced at the lower wall such that the normal velocity at the wall in the y – direction is equal to the blowing/suction control function. Therefore,

$$v|_{y=0} = v_w \qquad (7)$$

Where v_w is the control function. Since we are interested in controlling the wall shear stress, we define a cost function as follows:

$$J = \lim_{t_f \to \infty} \iiint_{t_i}^{t_f} \begin{bmatrix} \left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 \\ + \left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 \end{bmatrix}_{y=0} dzdxdt \qquad (8)$$

The cost functions serves the purpose of minimizing the control effort and keeps the system well within the region where the linear model is valid [5].

3. State Space Formulation

Next we discuss State Space formulation of the Physical system as described in reference [4]. The governing equations of the physical model can be represented in State Space form via Galerkin Method. Proper Orthogonal Decomposition (POD) is used to establish the correct basis functions. The resulting State Space equations obtained by reduced order modeling may then be written as:

$$\begin{bmatrix} \frac{dx_m}{dt} \\ \frac{dx_u}{dt} \end{bmatrix} = \begin{bmatrix} A_m & 0 \\ 0 & A_u \end{bmatrix}\begin{bmatrix} x_m \\ x_u \end{bmatrix} + \begin{bmatrix} B_m \\ B_u \end{bmatrix} u(t) \qquad (9)$$

$$z = [C_m \quad C_u]x(t) \qquad (10)$$

where the subscripts *m* and *u* represent the modeled and unmodelled dynamics of the system. Both the matrices A_m and A_u are infinite dimensional. The unmodelled part denotes the wave numbers left out of the finite dimensional model. The unmodelled wave number dynamics may be made uncontrollable by setting ***B*** matrix to zero. This simple means that the control input has no effect on the unmodelled wave number dynamics.

The cost function decouples into 2*nm* performance indices for each *nm* wave number pair.

$$\lim_{n \to \infty} \int_{t_i}^{t_f} \begin{bmatrix} R\hat{z}_{nm}^{*}\hat{z}_{nm} + \\ u_{nm}^{*} W_{nm}^{*} W_{nm} u_{nm} \end{bmatrix} dt \qquad (11)$$

4. Controller Design

It is proposed to design an Optimal Control Law on the lines of reference [5]. The internal state vector is obtained by an estimator (Kalman Filter) and Optimal Control is provided by Linear Quadratic Regulator (LQR) design which minimizes the cost function. The resulting LQG design can be represented by the following equations:

$$u = -\hat{K}_{nm}\tilde{x}_{nm} \qquad (12)$$

$$\frac{d\tilde{x}_{nm}}{dt} = \hat{A}_{nm}\tilde{x}_{nm} + \hat{B}_{nm}u_{nm} + \hat{L}_{nm}\left[z_{nm} - \hat{C}_{nm}\tilde{x}_{nm} - D_{nm}u_{nm}\right] \qquad (13)$$

where $-\hat{K}_{nm}$ is the gain matrix obtained by minimizing the performance index and $\hat{L}_{nm}$ is the Kalman Filter gain. The power spectral densities are chosen in such a way to keep the estimator eigenvalues same as the controller eigenvalues.

The input to the controller are the velocity vector gradients at the lower wall (y = 0) and the output matrix is forcing function given as blowing / suction. The wall fluctuations are sensed by a two dimensional FFT algorithm and converted into z_{nm}. An inverse FFT converts the u_{nm} into the columns of the matrix containing the blowing / suction at the wall. The entire routine can be plugged into a time dependent Navier – Stokes solver which gives good resolution at near wall boundaries for capturing the shear stress variations.

5. Conclusions

This article presented a framework for designing control laws to stabilize a fluid system by reduced order modeling of the system dynamics which looks promising in developing real world solutions for control of transition to turbulent flows and control of fluid flows in general. The challenges in this scope of work are the capability to handle higher dimensional models of the system by designing multiple sensor / actuator pairs for each modeled wave number and design of efficient CFD codes for capturing the near wall physics.

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