*International Journal of Advanced Computer Research (ISSN (print): 2249-7277 ISSN (online): 2277-7970)*
*Volume-2 Number-3 Issue-5 September-2012*

# Reduction of Noise Image Using LMMSE

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## Abstract

***There exist various image de-noising techniques. Amongst them orthogonal wavelet is preferred one. However, the orthogonal wavelet transform is not better technique as proper clustering of wavelet coefficients is not possible in this technique. So a better image de-noising technique is needed to have a better SNR and greater image information. In this work, image de-noising by linear minimum mean square-error estimation (LMMSE) scheme is proposed and results show that this method outperforms some of the existing de-noising techniques.***

## Keywords

***Noise reduction of image, LMMSE, wavelet transforms, optimal wavelet.***

## 1. Introduction

Stastical modeling is very important for the effectiveness of signal processing. A wavelet transform (WT), can decorrelate random processes into independent coefficients, which can then be more effectively modeled statistically [7, 8, 19, 23, 24]. WT can be successfully applied to coding and denoising. The first wavelet soft thresholding approach by Donoho and many wavelet-based denoising schemes are reported [2, 3, 5, 9, 12-28]. In threshold-based denoising schemes, a threshold is set to distinguish noise from the structural information. Thresholding can be classified into soft and hard ones, in which coefficients less than the threshold will be set to 0 but those above the threshold will be preserved. Donoho [2] first presented the wavelet shrinkage scheme with a universal threshold based on orthonormal wavelet bases. Since Donoho's pioneer work, a numerous threshold-based denoising schemes have been proposed [3, 13, 17-21]. It is generally accepted that in each sub-band the image wavelet coefficients can be modelled as independent identically distributed random variables with generalized Gaussian distribution (GGD) with which Chang presented a near optimal soft threshold [20-23, 26].

Liu and Moulin [11] classified the wavelet statistical models into intrascale, interscale and hybrid ones. The denoising schemes in [16-26] benefit from intrascale models. Chang et al. [21] introduced a spatially adaptive wavelet thresholding scheme based on context modeling. M. K. Mıhçak et al. [16] estimated the second-order local statistics of each coefficient with a centered square-shaped window and developed linear minimum mean squared-error estimation (LMMSE) like denoising method. The denoising approach of Li and Orchard [26] is also LMMSE based but it models the wavelet coefficients as a mixture of edge and non-edge classes. In [5], a local contextual hidden Markov model (LCHMM) was proposed to capture the wavelet intrascale dependencies. Wavelet interscale models are also used in many other applications [1, 6, 10, 13-15, 17, 28]. Shapiro [10] exploited this property and developed the well-known embedded zero tree wavelet image compression scheme. The property has been exploited for denoising [13, 17, 28] step estimation and edge detection. The wavelet interscale dependencies have also been represented by Markov models. Each coefficient was modelled as the product of a Gaussian random vector and a hidden multiplier variable to include adjacent scales in the conditioning local neighbourhood [6, 9, 11, 12, 15, and 26].The rest of this paper is organized as follows: In Section 2, details of methodology are formulated. Section 3 deals with the results and discussion. Finally, the concluding remarks are given in Section 4.

## 2. Methodologies

The LMMSE denoising schemes in and exploits the wavelet intrascale dependencies [16, 26]. An LMMSE-based denoising approach with an interscale model is presented by using over complete wavelet expansion (OWE). We have exploited the wavelet intrascale dependency to spatially classify the wavelet coefficients into several clusters adaptively. With OWE, in which there is no down sampling in the decomposition, each wavelet subband has the same number of coefficients as the input image. We combine the wavelet coefficients with the same spatial location across adjacent scales as a vector, to which the LMMSE is then applied. Such an operation naturally incorporates the interscale dependencies of wavelet coefficients to improve the estimation. LMMSE is similar to soft thresholding strategy. Suppose the variable is scalar, instead of shrinking a noisy wavelet coefficient $w = x + v$ (where $x$ is the wavelet coefficient of noiseless signal and $v$ is that of noise) with threshold $t : \hat{x} = \mathrm{sgn}(w) \cdot \max(|w - t, 0)$, LMMSE modifies the coefficient with a factor$c$ : $\hat{x} = c \cdot w$, where $c = \sigma_x^2/(\sigma_x^2 + \sigma_v^2)$ and $\sigma_v^2$ are the variances of signal and $v$ noise, respectively. Obviously, is less than 1 so that $|\hat{x}|$ will be less than $|w|$ .The energy of finally restored signal will be shrunk just likes in the soft thresholding schemes. The performance of proposed interscale LMMSE scheme is wavelet dependent [7, 8].

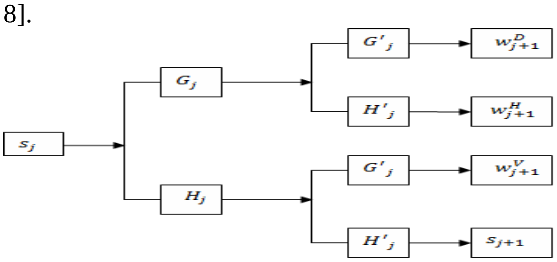


**Fig. 1: One stage decomposition of the 2-D OWE.**

$w_j^H$ , $w_j^V$ and $w_j^D$ are the wavelet coefficients at horizontal, vertical and diagonal directions. From denoising point of view wavelet filters should have the following two properties. One is the capability of extracting signal information from noisy wavelet coefficients. A parameter M, which is based on the mutual information of noiseless wavelet coefficients and noisy wavelet coefficients, is defined M is proportional to the performance of the scheme. The other is that the distribution of interscale image wavelet coefficients is sufficiently close to jointly Gaussian. A parameter, which measures the difference between the Gaussian and real signal density functions, is defined and is inversely proportional to the denoising performance. An optimal wavelet could be determined from a library of wavelets based on the M and E values [26].

Use of context modelling gives a local discrimination of image characteristics, such as edge structures and backgrounds, according to their spatial dependencies. We extend the context modelling to interscale wavelet coefficient vector variables. The statistics of wavelet coefficients are then estimated locally from each cluster. Experiments show that context modelling improves the denoising performance [26].

### 2.1. Interscale model and LMMSE-based denoising

Bi-orthogonal wavelet transform (OWT) is translation variant due to the down sampling. This will cause some visual artifacts in threshold-based denoising. It has been observed that the OWE achieves better results in noise reduction and artifacts suppression. The denoising scheme presented adopts OWE, whose one stage two-dimensional (2-D) decomposition structure is shown in Fig. 1[courtesy from ref. 26]. The restored signal by OWE is an average of several circularly shifted denoised versions of the same signal by OWT, and by which the additive noise is better suppressed [17, 18, 20, 26].

### 2.2. LMMSE of wavelet coefficients

Let the original [26] signal f is corrupted with additive Gaussian white noise $\varepsilon$

$$g = f + \varepsilon \qquad (1)$$

where $\varepsilon \in N(0, \sigma^2)$ . Applying the OWE to the noisy signal g, at scale j gives

$$w_j = x_j + v_j \qquad (2)$$

where $w_j$ is coefficients at scale j, $x_j$, and $v_j$ are the expansions of $f$ and $\varepsilon$ , respectively. Here, the LMMSE of wavelet coefficients is employed instead of soft thresholding. Suppose the variance of $v_j$ is $\sigma_j^2$ and that of $x_j$ is $\sigma_{x_j}^2$ . Since both are zero mean, the LMMSE of $x_j$ is

$$\hat{x}_j = c' \cdot w_j \qquad (3)$$

with

$$c = \frac{\sigma_{x_j}^2}{\sigma_j^2 + \sigma_{x_j}^2} \qquad (4)$$

Since $v_j$ is Gaussian distributed and independent of $x_j$ , if $x_j$ is also of Gaussian distribution, it is well known that $w_j$ will be Gaussian and (3) is equivalent to the optimal MMSE [4]. Unfortunately, $x_j$ obeys in general the GGD model, which reduces to Gaussian only in very special cases.

Referring to Fig. 1, term $w_{j+1}^D$ can be written as

$$w_{j+1}^H = s_0 * L_j^H \qquad (5)$$

where $*$ is the convolution operator and filter $L_j^D$ is

$$L_j^D - H_0 * H'_0 * \cdots * H_{j-1} * H'_{j-1} * G_j * G'_j \qquad (6)$$

Similarly, we have

$$w_{j+1}^H = s_0 * L_j^H \ , \ w_{j+1}^V = s_0 * L_j^V \qquad (7)$$

where

$$L_j^H = H_0 * H'_0 * \cdots * H_{j-1} * H'_{j-1} * G_j * H'_j \quad (8)$$

$$L_j^V = H_0 * H'_0 * \cdots * H_{j-1} * H'_{j-1} * H_j * G'_j \qquad (9)$$

Noise standard deviation of $v_j$ at scale j in a direction (horizontal, vertical or diagonal) is

$\sigma_j = \|L_{j-1}\|\sigma$ (10) where $L_{j-1}$ is the corresponding filter ($L_{j-1}^{D}$ , $L_{j-1}^{H}$ or $L_{j-1}^{V}$ ) and $\|\cdot\|$ is the norm operator: $\|L\| = \sqrt{\sum_l \sum_k L^2(l,k)}$. The standard deviation $\sigma_{x_j}^2$ of noiseless image $x_j$ is estimated as follows

$$\hat{\sigma}_{x_j}^2 = \sigma_{w_j}^2 - \sigma_j^2 \quad (11)$$

With

$$\sigma_{w_j}^2 = \frac{1}{M \cdot N} \sum_{m=1}^{M} \sum_{n=1}^{N} w_j^2 (m,n) \quad (12)$$

Where M and N are the numbers of input image rows and columns. LMMSE is similar to soft thresholding in some sense. Notice that factor c is always less than 1, thus the magnitude of estimated wavelet coefficient $\hat{x}_j$ would be less than that of $w_j$ . This leads to the energy shrinkage of the restored signal, likewise in the soft thresholding schemes [26].

### 2.3. Interscale wavelet model-based LMMSE

we would make no use of the measurements at the finer scale to estimate the signal at the coarser scale, and $x_j$ is estimated only by measurements at scales j and $j+1$ . We assemble the points with the same orientation at scales j and $j+1$ as a vector

$$\overrightarrow{w_j}(m,n) = \left[w_j(m,n)\ w_{j+1}(m,n)\right]^T \quad (13)$$

Thus

$$\overrightarrow{w_j} = \overrightarrow{x_j} + \overrightarrow{v_j} \quad (14)$$

With $\overrightarrow{x_j}(m,n) = \left[x_j(m,n)\ \ x_{j+1}(m,n)\right]^T$

$$\overrightarrow{v_j}(m,n) = \left[v_j(m,n)\ \ v_{j+1}(m,n)\right]^T \quad (15)$$

$\overrightarrow{v_j}$ is a Gaussian noise vector independent of $\overrightarrow{x_j}$. The LMMSE of $\overrightarrow{x_j}$ is then

$$\hat{\overrightarrow{x_j}} = P_j\left(P_j + R_j\right)^{-1}\overrightarrow{w_j} \quad (16)$$

where $P_j$ and $R_j$ are the covariance matrices of $\overrightarrow{x_j}$ and $\overrightarrow{v_j}$ , respectively

$$P_j = E\left[\overrightarrow{x_j}\,\overrightarrow{x_j}^T\right] = E\begin{bmatrix} x_j^2 & x_j x_{j+1} \\ x_j x_{j+1} & x_{j+1}^2 \end{bmatrix}$$

$$R_j = E\left[\overrightarrow{v_j}\ \overrightarrow{v_j}^T\right] = E\begin{bmatrix} v_j^2 & v_j v_{j+1} \\ v_j v_{j+1} & v_{j+1}^2 \end{bmatrix} \quad (17)$$

Let us compute the components of noise covariance matrix $R_j$ first. The diagonal element $E\left[v_j^2\right]$ is equal to $\sigma_j^2$ which can be obtained by [3]. Noise variables $v_j$ and $v_{j+1}$ are the projections of v on different wavelet subspaces. They are correlated with correlation coefficient

$$\rho_{j,j+1} = \frac{\sqrt{\sum_l \sum_k L_{j-1}(l,k)\, L_j\ (l,k)}}{\|L_{j-1}\| \cdot \|L_j\|} \quad (18)$$

$v_j$ and $v_{j+1}$ are jointly Gaussian and their density is$\left(v_j, v_{j+1}\right) =$

$$\frac{1}{2\pi\sqrt{1-p_{j,j+1}^2}\ \ \sigma_j\sigma_{j+1}} \times e^{-\frac{1}{2\left(1-p_{j,j+1}^2\right)}\left[\frac{v_j^2}{\sigma_j^2} - \frac{2v_{j+1}\,v_j p_{j,j+1}}{\sigma_{j+!}\sigma_j} + \frac{v_{j+1}^2}{\sigma_{j+1}^2}\right]} \quad (19)$$

Thus, the expectation $E\left[v_j v_{j+1}\right]$ is

$$E\left[v_j v_{j+1}\right] = p_{j,j+1}\sigma_j\sigma_{j+1} \quad (20)$$

Each of the components of matrix $P_j$ is estimated by

$$E[x_l x_k] \approx E[w_l w_k] - E[v_l v_k] \quad (21)$$

where $l,k = j, j+1$ and $E[w_l w_k]$ is computed as

$$E[w_l w_k] = \frac{1}{M \cdot N} \sum_{m=1}^{M} \sum_{n=1}^{N} w_l\,(m,n) \quad (22)$$

After the LMMSE result $\hat{\hat{x}}_j$ is obtained, only the component $\hat{x}_j$ is extracted. Estimation of $\hat{x}_{j+1}$ would be obtained from the LMMSE result $\hat{\hat{x}}_{j+1}$ [26].

### 2.4. Optimal wavelet basis selection

The denoising performance of the proposed LMMSE-based scheme varies with different wavelet filters. Ideally, a good wavelet filter for denoising should meet the following two requirements. One is the interscale model's ability in extracting signal information from noisy wavelet coefficients. The other is a high degree of agreement between the distribution of wavelet coefficients and Gaussian distribution.

#### 2.4.1. Signal information extraction criterion

The mutual information [26] of $\mu$ and v is defined as $I(\mu, v) = \sum_\mu \sum_v p(\mu, v) \log \frac{p(\mu,v)}{p(\mu)p(v)}$ (23)

The higher $I(\mu, v)$ is, the more information $\mu$ could provide to estimate v or vice-versa. If $\mu$ is a function of v , $I(\mu, v)$ will be infinite. Otherwise, if $\mu$ is independent with v, obviously $I(\mu, v)$ is zero.We take the mutual information of $\overrightarrow{x_j}$ and $\overrightarrow{w_j}$ as a measure to evaluate how much signal information could be exploited $\overrightarrow{w_j}$ from to estimate $\overrightarrow{x_j}$ .We have derived that $\overrightarrow{v_j}$ is Gaussian with covariance matrix $R_j$. The covariance matrix of $\overrightarrow{x_j}$ is $P_j$ and we assume $\overrightarrow{x_j}$ is also Gaussian. Since $\overrightarrow{w_j} = \overrightarrow{x_j} + \overrightarrow{v_j}$ , the mutual information of $\overrightarrow{w_j}$ and $\overrightarrow{x_j}$ is [25]

$M_j = I\left(\overrightarrow{x_j}, \overrightarrow{w_j}\right) = \frac{1}{2}\log\left(\frac{|P_j+R_j|}{|R_j|}\right)$ (24) where $|\cdot|$ represents the determinant of a matrix. The criterion $M_j$ is proportional to the performance of the proposed denoising scheme. A properly selected wavelet should yield a significant value of $M_j$ , which means noisy coefficients $\overrightarrow{w_j}$ could give significant information to estimate original signal $\overrightarrow{x_j}$. Since the image wavelet coefficients are subjected to GGD, the distribution of $\overrightarrow{x_j}$ would be of some difference with bivariate Gaussian function. The errors so caused could be generalized into the following criterion [26].

### 2.4.2. Distribution error criterion

The distribution of wavelet coefficients is often modeled as GGD [20]

$$GG_{\beta,\sigma_x}(x) = C(\beta,\sigma_x)e^{-(\alpha(\beta,\sigma_x)|x|)^{B}}, -\infty < x < \infty, \sigma_x > 0, \beta > 0 \quad (25)$$

$$\alpha(\beta,\sigma_x) = \sigma_x^{-1}\left[\frac{\Gamma\left(\frac{3}{\beta}\right)}{\Gamma\left(\frac{1}{\beta}\right)}\right]^{\frac{1}{2}} \quad C(\beta,\sigma_x) = \frac{\beta\alpha(\beta,\sigma_x)}{2\Gamma\left(\frac{1}{\beta}\right)} \quad (26)$$

where $\sigma_x$ is the standard deviation of x, $\beta$ is the shape parameter and $\Gamma(t) = \int_0^{\infty} e^{-u}u^{t-1}\,du$ is the Gamma function. GGD is zero-mean and degenerates to Gaussian distribution only when $\beta = 2$. The Gaussian function

$$G_{\sigma_x} = \frac{1}{\sqrt{2\pi\sigma_x}}e^{\frac{-x^2}{2\sigma_x^2}} \quad (27)$$

$$p_g(x_j, x_{j+1}) = \frac{1}{2\pi\sigma_{x_j}\sigma_{x_{j+1}}\sqrt{1-p_j^2}} \times e^{\frac{-1}{2(1-p_j^2)}\left[\frac{x_j^2}{\sigma_{x_j}^2} - \frac{2p_jx_jx_{j+1}}{\sigma_{x_j}\sigma_{x_{j+1}}} - \frac{x_{j+1}^2}{\sigma_{x_{j+1}}^2}\right]} \quad (28)$$

where $p_j$ is calculated as

$$p_j = \frac{E[x_jx_{j+1}]}{\sigma_{x_j}\sigma_{x_{j+1}}} = \frac{1}{M\cdot N\cdot\sigma_{x_j}\sigma_{x_{j+1}}} \times \sum_{m=1}^{M}\sum_{n=1}^{N} x_j(m,n)\cdot x_{j+1}(m,n) \quad (29)$$

We define the distribution error criterion as a kind of Hellinger distance $E_j = \sqrt{\int\int(p_g - p)^2 dx_j}$ (30)

When p and $p_g$ are identical, the measurement $E_j$ will reach the minimum 0. The higher the error $\tilde{p} = p - p_g$, the higher the value of $E_j$, which implies that $p_g$ worse approximates a joint Gaussian distribution, and then the LMMSE will be much inferior to the MMSE. So a good wavelet should yield a small $E_j$ [26].A block diagram of proposed modeling is shown in Fig. 2.

## 3. Results and Discussions

This section compares the results from different wavelet for proposed scheme in terms of SNR. The noisy images are simulated by adding Gaussian white noise on the original images. In threshold-based (hard or soft) de-noising schemes, the wavelet coefficients whose magnitudes are below a threshold will be set to 0. The corresponding pixels are generally noise predominated and thus the thresholding of these coefficients is safely a structure-preserving de-noising process. We apply the LMMSE only to those coefficients above a threshold and shrink those below the threshold to 0. It should be noted that the images used here are 256 x256, while the images used in 750x550x3. At the same noise level, the denoising results of high resolution images are much better than those of low resolution images. Fig 3 shows original images and Fig 4 represents images after noise addition .De-noised images are shown in Fig 5 and the comparisons of SNR ratios and mean square errors are given in Table 1.

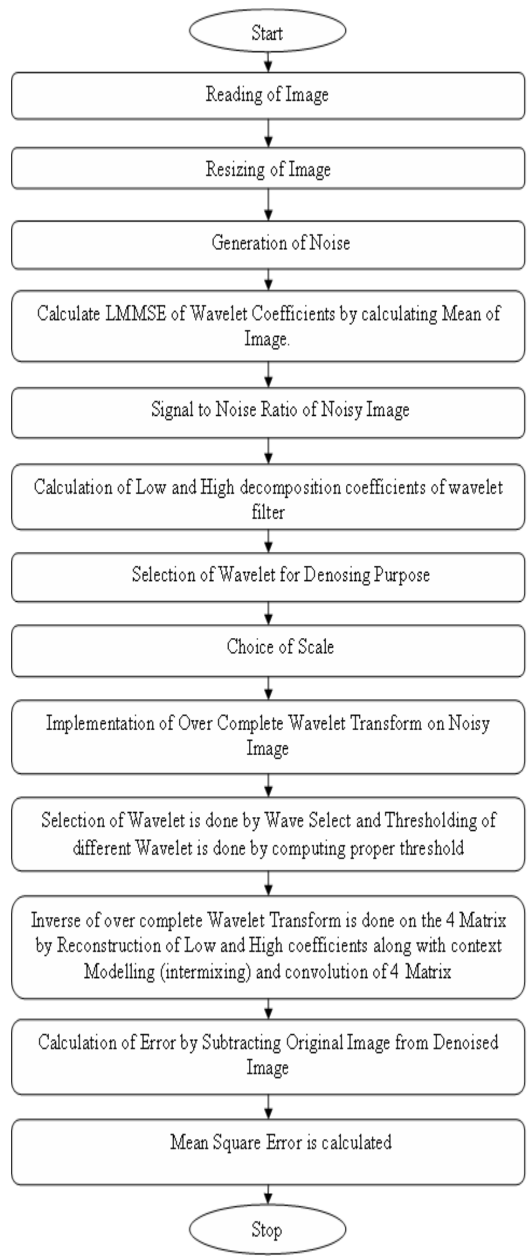


**Fig. 2: Proposed modeling.**

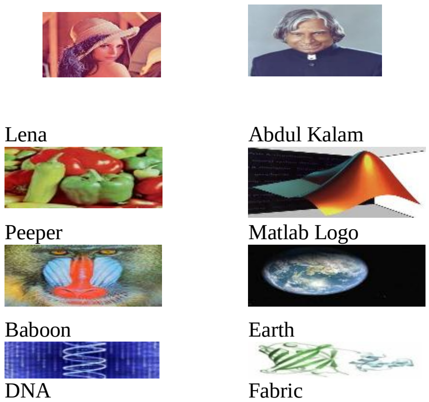


**Fig. 3: Original images**

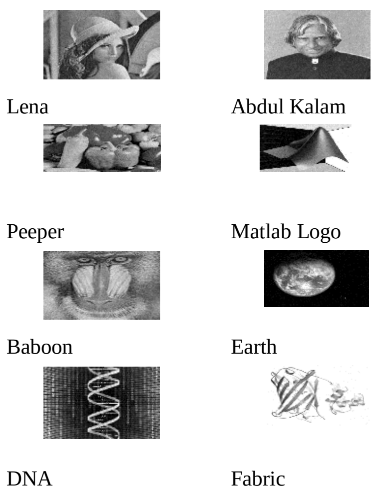


**Fig. 4: Images after noise addition.**

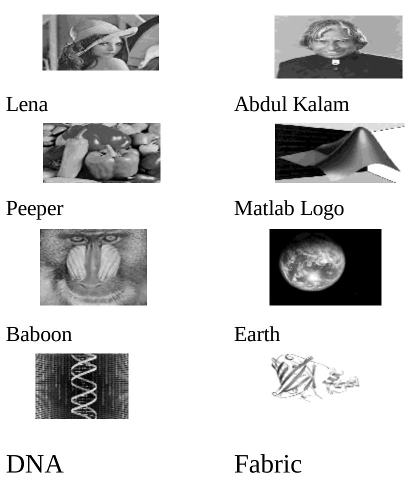


**Fig. 5: De-noised images.**

**Table 1: Comparisons of SNR ratios and mean square errors.**

| Name | Original (SNR) | Noisy Image (SNR) | De-Noisy Image (SNR) | Mean Sq. Error |
|---|---|---|---|---|
| Baboon | 36.05 | 32.9016 | 36.0665 | 48.40 |
| Lena | 37.27 | 32.4739 | 37.2572 | 33.14 |
| DNA | 31.72 | 29.3680 | 31.7399 | 58.22 |
| Abdul Kalam | 39.55 | 33.0122 | 39.6156 | 22.21 |
| Logo Matlab | 40.04 | 33.4299 | 40.0251 | 21.84 |
| Peeper | 36.16 | 32.0965 | 36.2385 | 36.26 |
| Earth | 34.33 | 28.1389 | 34.3622 | 24.05 |
| Fabric | 42.11 | 37.3835 | 42.1257 | 33.64 |

## 4. Conclusions

Wavelet-based LMMSE scheme for image denoising along with OWE is used for determination of the optimal wavelet basis. To explore the strong inter-scale dependencies of OWE, we combine the pixels at the same spatial location across scales as a vector and apply LMMSE to the vector. Compared with the LMMSE within each scale, the inter-scale model exploits the dependency information distributed at adjacent scales. The performance of the proposed scheme is dependent on the selection of the wavelet bases. Two criteria, the signal information extraction criterion and the distribution error criterion, are proposed to measure the de-noising performance. The optimal wavelet that achieves the best tradeoff between the two criteria can be determined from a library of wavelet bases. Experiments show that the proposed scheme outperforms some of the existing de-noising techniques.

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