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Estimation of Variance of the time to recruitment under two sources of depletion

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Abstract

In this paper, the problem of time to recruitment is studied using a univariate policy of recruitment for a two grade manpower system in which attrition which leads to loss of manpower takes place due to two sources of depletion. A stochastic model is constructed and the variance of the time to recruitment is obtained when the inter-policy decision times for the two grades form same renewal processes but the inter-transfer decision times form the two different renewal processes. The results are illustrated and specific conclusions are given on the influence of parameters over the performance measures.

Keywords

Two grade manpower system, two sources of depletion, univariate recruitment policy, renewal process and variance of the time to recruitment

1. Introduction

Many researchers [1], [2], [3], [4] and [5] have studied the problem of time to recruitment for a marketing organization consisting of two grades by considering ***only one source*** (recurring) for depletion of man power which takes place due to attrition as the effect of policy decisions such as revision of pay, targets etc. taken by the organization, using univariate and bivariate policies of recruitment under different conditions. In this context, Suresh Kumar et.al [6] have obtained the mean and variance of the time to recruitment for a two grade manpower system when the exponential inter-decision times for the two grades form two different renewal processes and the threshold for the loss of man power in the organization is minimum of the exponential thresholds for the loss of manpower in the two grades.

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Apart from the above cited source of depletion which is recurring there is another source of depletion due to transfer decisions which is non-recurrent. In the presence of these ***two different sources*** of depletion, Elangovan et.al [7] have studied the problem of time to recruitment for an organization consisting of one grade and obtained the variance of the time to recruitment using a univariate policy of recruitment when (i) the loss of man power in the organization due to the two sources of depletion and its threshold are independent and identically exponential random variables. (ii) Inter-policy decision times and inter-transfer decision times form the same renewal process. Recently, Dhivya and Srinivasan [8] have studied the work of Elangovan et.al for two grade system and obtained several performance measures by considering different forms of the loss of manpower and its threshold for the organization. The present paper studies the problem of time to recruitment for a two grade manpower system with two sources of depletion when the renewal processes governing the inter-policy decisions times for the two grades are same but the renewal processes governing the inter-transfer decisions times for the two grades are different.

2. Model Description

Consider an organization taking decisions at random epoch (0, ∞) and at every decision making epoch a random number of persons quit the organization. Each exit of personnel produces a loss in manpower which is linear and cumulative. For i=1,2,3…, let X_{Ai} and X_{Bi} be the continuous random variables representing the amount of depletion of manpower (loss of man hours) in grades A and B respectively caused due to the i^{th} policy decision. It is assumed that X_{Ai} and X_{Bi} are independent for each i and each form a sequence of independent and identically distributed random variables with probability density functions $g_A(.)$ and $g_B(.)$ respectively. Write $\tilde{X}_{Am} = \sum_{j=1}^{m} X_{Ai}$ and $\tilde{X}_{Bm} = \sum_{j=1}^{m} X_{Bi}$. For j=1,2,3…, let Y_{Aj} and Y_{Bj} be the continuous random variables representing the amount of depletion of manpower in grades A and B respectively caused due to the j^{th}

transfer decision. It is assumed that Y_{Aj} and Y_{Bj} are independent for each j and each form a sequence of independent and identically distributed random variables with probability density functions $h_A(.)$ and $h_B(.)$ respectively. Write $\tilde{Y}_{An_1} = \sum_{j=1}^{n_1} Y_{Aj}$ and $\tilde{Y}_{An_2} = \sum_{j=1}^{n_2} Y_{Bj}$. For each i and j X_{Ai}, X_{Bi}, Y_{Aj} and Y_{Bj} are statistically independent. Let $\bar{s}_k(.)$ be the Laplace transforms of $s_k(.)$. Let Z_A and Z_B be independent exponentially distributed threshold levels for the depletion of manpower in grades A and B with mean $\frac{1}{\theta_A}$ and $\frac{1}{\theta_B}(\theta_A, \theta_B > 0)$ respectively and let Z be the threshold level for the depletion of manpower in the organization with probability density function $k(.)$. It is assumed that the number of policy decisions form a the same renewal process for the both the grades governed by a sequence of independent and identically distributed exponential inter-decision times with distribution $F(.)$, probability density function $f(.)$ and mean $\frac{1}{\mu_1}$ $(\mu_1 > 0)$ respectively . Let the inter-transfer decision times for grades A and B be independent and identically distributed exponential random variables with distribution $W(.)$ and V(.), probability density function $w(.)$ and v(.) and mean $\frac{1}{\mu_{2A}}$ and $\frac{1}{\mu_{2B}}$ $(\mu_{2A}, \mu_{2B} > 0)$ respectively. It is assumed that the two sources of depletion are independent. Let $C_m(.)$ be the m-fold convolution of $C(.)$ with itself. The univariate CUM policy of recruitment employed in this paper is stated as follows:

Recruitment is done whenever the cumulative loss of man hours in the organization exceeds the threshold for the loss of man hours in this organization.

Let T be the random variable denoting the time to recruitment with distribution L(.), mean E(T) and variance V(T). Let $N_p(T)$ be the number of policy decisions and $N_{Trans.}(T)$ be the number of transfer decisions taken in (0,T). Let $\bar{X}_{N_p(T)}$ and $\bar{Y}_{N_{Trans.}(T)}$ be the respective total loss of manpower in $N_p(T)$ and $N_{Trans.}(T)$ decisions incurred by the organization during (0,T).

Main Results

The tail distribution of T is given by

$$P(T>t) = \begin{Bmatrix} \text{Probability that there are excatly m policy decisions in both grades} \\ n_1 \text{transfer decision in grade A and } n_2 \text{transfer decision in grade B} \\ \text{and total loss of manpower does not crosses the threshold Z} \end{Bmatrix}$$

$$P(T>t) = \sum_{m=0}^{\infty}\sum_{n_1}^{\infty}\sum_{n_2=0}^{\infty}[F_m(t)-F_{m+1}(t)][W_{n_1}(t)-W_{n_1+1}(t)]$$
$$[V_{n_2}(t)-V_{n_2+1}(t)]P(\tilde{X}_{mA}+\tilde{X}_{mB}+\tilde{Y}_{n_1A}+\tilde{Y}_{n_2B}<Z) \quad (1)$$

Invoking to the law of total probability, we get

$$P(\tilde{X}_{mA}+\tilde{X}_{mB}+\tilde{Y}_{n_1A}+\tilde{Y}_{n_2B}<Z)$$
$$= \int_0^{\infty} P(\tilde{X}_{mA}+\tilde{X}_{mB}+\tilde{Y}_{n_1A}+\tilde{Y}_{n_2B}<z)\; k(z)\,dz \qquad (2)$$

By considering three kinds for the threshold Z, explicit expressions for E(T) and V(T) are obtained below with the help of (1) and (2).

Case (i) $Z = \min(Z_A, Z_B)$

In this case it can be shown that

$$k(z) = (\theta_A+\theta_B)e^{-(\theta_A+\theta_B)z} \qquad (3)$$

From (1),(2)and (3)

$$P(T>t) = \sum_{m=0}^{\infty}[F_m(t)-F_{m+1}(t)][\bar{g}_A(\theta_A+\theta_B)\bar{g}_B(\theta_A+\theta_B)]^m$$
$$\sum_{n_1=0}^{\infty}[W_{n_1}(t)-W_{n_1+1}(t)][\bar{h}_A(\theta_A+\theta_B)]^{n_1}$$
$$\sum_{n_2=0}^{\infty}[V_{n_2}(t)-V_{n_2+1}(t)][\bar{h}_B(\theta_A+\theta_B)]^{n_2}$$

On simplification, it can be shown that

$$P(T>t) = \{1-[1-\bar{g}_A(\theta_A+\theta_B)\bar{g}_B(\theta_A+\theta_B)]$$
$$\sum_{m=1}^{\infty}F_m(t)[\bar{g}_A(\theta_A+\theta_B)\bar{g}_B(\theta_A+\theta_B)]^{m_1-1}\}$$
$$\{1-[1-\bar{h}_A(\theta_A+\theta_B)]\sum_{n_1=1}^{\infty}W_{n_1}(t)[\bar{h}_A(\theta_A+\theta_B)]^{n_1-1}\}$$
$$\{1-[1-\bar{h}_B(\theta_A+\theta_B)]\sum_{n_2=1}^{\infty}V_{n_2}(t)[\bar{h}_B(\theta_A+\theta_B)]^{n_2-1}\} (4)$$

Since $f_m(t) = \frac{\mu_1{}^m e^{-\mu_1 t} t^{m-1}}{(m-1)!}$, $w_{n_1}(t) = \frac{\mu_{2A}{}^{n_1} e^{-\mu_{2A} t} t^{n_1-1}}{(n_1-1)!}$, and $v_{n_2}(t) = \frac{\mu_{2B}{}^{n_2} e^{-\mu_{2B} t} t^{n_2-1}}{(n_2-1)!}$ by hypothesis, we find that

$$[1-\bar{g}_A(\theta_A+\theta_B)\bar{g}_B(\theta_A+\theta_B)]\sum_{m=1}^{\infty}F_m(t)[\bar{g}_A(\theta_A+\theta_B)\bar{g}_B(\theta_A+\theta_B)]^{m-1}$$
$$= 1-e^{-\mu_1[1-\bar{g}_A(\theta_A+\theta_B)\bar{g}_B(\theta_A+\theta_B)]t}$$
$$[1-\bar{h}_A(\theta_A+\theta_B)]\sum_{n_1=1}^{\infty}W_{n_1}(t)[\bar{h}_A(\theta_A+\theta_B)]^{n_1-1}$$
$$= 1-e^{-\mu_{2A}[1-\bar{h}_A(\theta_A+\theta_B)]t}$$
$$[1-\bar{h}_B(\theta_A+\theta_B)]\sum_{n_2=1}^{\infty}V_{n_2}(t)[\bar{h}_B(\theta_A+\theta_B)]^{n_2-1}$$
$$= 1-e^{-\mu_{2B}[1-\bar{h}_B(\theta_A+\theta_B)]t} \qquad (5)$$

Therefore from (4) and (5) we get

$$L(t) = 1-e^{-[\mu_1[1-\bar{g}_A(\theta_A+\theta_B)\bar{g}_B(\theta_A+\theta_B)]+\mu_{2A}[1-\bar{h}_A(\theta_A+\theta_B)]+\mu_{2B}[1-\bar{h}_B(\theta_A+\theta_B)]]t} \quad (6)$$

which is exponential distribution with parameter

$\mu_1[1-\bar{g}_A(\theta_A+\theta_B)\bar{g}_B(\theta_A+\theta_B)]+\mu_{2A}[1-\bar{h}_A(\theta_A+\theta_B)]+\mu_{2B}[1-\bar{h}_B(\theta_A+\theta_B)]$

We now obtain several performance measures from (6).

From (6) we get

$$E(T)=\frac{1}{\mu_1[1-\bar{g}_A(\theta_A+\theta_B)\bar{g}_B(\theta_A+\theta_B)]+\mu_{2A}[1-\bar{h}_A(\theta_A+\theta_B)]+\mu_{2B}[1-\bar{h}_B(\theta_A+\theta_B)]}$$

$$V(T)=\frac{1}{\left[\mu_1[1-\bar{g}_A(\theta_A+\theta_B)\bar{g}_B(\theta_A+\theta_B)]+\mu_{2A}[1-\bar{h}_A(\theta_A+\theta_B)]+\mu_{2B}[1-\bar{h}_B(\theta_A+\theta_B)]\right]^2}$$

Hazard rate at t $=$
$\mu_1[1-\bar{g}_A(\theta_A+\theta_B)\bar{g}_B(\theta_A+\theta_B)]+\mu_{2A}[1-\bar{h}_A(\theta_A+\theta_B)]+\mu_{2B}[1-\bar{h}_B(\theta_A+\theta_B)]$

Probability that recruitment takes place in (t, t+dt) given that there is no recruitment in [0,t]
$=P(t<T<t+dt/T>t)$
$=[1-e^{-[\mu_1[1-\bar{g}_A(\theta_A+\theta_B)\bar{g}_B(\theta_A+\theta_B)]+\mu_{2A}[1-\bar{h}_A(\theta_A+\theta_B)]+\mu_{2B}[1-\bar{h}_B(\theta_A+\theta_B)]]dt}]$

Average residual time for recruitment given that there is no recruitment upto time t.
$=E(T-t/T>t)=$

$$\frac{1}{\mu_1[1-\bar{g}_A(\theta_A+\theta_B)\bar{g}_B(\theta_A+\theta_B)]+\mu_{2A}[1-\bar{h}_A(\theta_A+\theta_B)]+\mu_{2B}[1-\bar{h}_B(\theta_A+\theta_B)]}$$

Again from (6) one can prove the following results using law of total probability for expectation.

Average number of policy and transfer decisions required to make recruitment at T
$=(\mu_1+\mu_{2A}+\mu_{2B})E(T)$

Average total loss of manpower due to $\mathbf{N_P(T)}$ and $\mathbf{N_{Trans.}(T)}$ decisions
$=\{\mu_1[E(X_{Ai})+E(X_{Bi})]+\mu_{2A}E(Y_{Aj})+\mu_{2B}E(Y_{Bj})\}E(T)$

Case (ii) Let $Z=\max(Z_A,Z_B)$

For this case,

$$k(z)=\left[\theta_A e^{-\theta_A z}+\theta_B e^{-\theta_B z}-(\theta_A+\theta_B)e^{-(\theta_A+\theta_B)z}\right] \qquad (7)$$

From (1),(2)and (7)

$$\begin{aligned}P(T>t)=&\sum_{m=0}^{\infty}[F_m(t)-F_{m+1}(t)]\,[\bar{g}_A(\theta_A)\bar{g}_B(\theta_A)]^m\\&\sum_{n_1=0}^{\infty}[W_{n_1}(t)-W_{n_1+1}(t)]\,[\bar{h}_A(\theta_A)]^{n_1}\sum_{n_2=0}^{\infty}[V_{n_2}(t)-V_{n_2+1}(t)]\,[\bar{h}_B(\theta_A)]^{n_2}\\&+\sum_{m=0}^{\infty}[F_m(t)-F_{m+1}(t)]\,[\bar{g}_A(\theta_B)\bar{g}_B(\theta_B)]^m\\&\sum_{n_1=0}^{\infty}[W_{n_1}(t)-W_{n_1+1}(t)]\,[\bar{h}_A(\theta_B)]^{n_1}\sum_{n_2=0}^{\infty}[V_{n_2}(t)-V_{n_2+1}(t)]\,[\bar{h}_B(\theta_B)]^{n_2}\\&-\sum_{m=0}^{\infty}[F_m(t)-F_{m+1}(t)]\,[\bar{g}_A(\theta_A+\theta_B)\bar{g}_B(\theta_A+\theta_B)]^m\\&\sum_{n_1=0}^{\infty}[W_{n_1}(t)-W_{n_1+1}(t)]\,[\bar{h}_A(\theta_A+\theta_B)]^{n_1}\sum_{n_2=0}^{\infty}[V_{n_2}(t)-V_{n_2+1}(t)]\,[\bar{h}_B(\theta_A+\theta_B)]^{n_2}\end{aligned}$$

On simplification

$$\begin{aligned}P(T>t)=&\left\{1-[1-\bar{g}_A(\theta_A)\bar{g}_B(\theta_A)]\sum_{m=1}^{\infty}F_m(t)[\bar{g}_A(\theta_A)\bar{g}_B(\theta_A)]^{m-1}\right\}\\&\left\{1-[1-\bar{h}_A(\theta_A)]\sum_{n_1=1}^{\infty}W_{n_1}(t)[\bar{h}_A(\theta_A)]^{n_1-1}\right\}\left\{1-[1-\bar{h}_B(\theta_A)]\sum_{n_2=1}^{\infty}V_{n_2}(t)[\bar{h}_B(\theta_A)]^{n_2-1}\right\}\\&+\left\{1-[1-\bar{g}_A(\theta_B)\bar{g}_B(\theta_B)]\sum_{m=1}^{\infty}F_m(t)[\bar{g}_A(\theta_B)\bar{g}_B(\theta_B)]^{m-1}\right\}\\&\left\{1-[1-\bar{h}_A(\theta_B)]\sum_{n_1=1}^{\infty}W_{n_1}(t)[\bar{h}_A(\theta_B)]^{n_1-1}\right\}\left\{1-[1-\bar{h}_B(\theta_B)]\sum_{n_2=1}^{\infty}V_{n_2}(t)[\bar{h}_B(\theta_B)]^{n_2-1}\right\}\\&-\left\{1-[1-\bar{g}_A(\theta_A+\theta_B)\bar{g}_B(\theta_A+\theta_B)]\sum_{m=1}^{\infty}F_m(t)[\bar{g}_A(\theta_A+\theta_B)\bar{g}_B(\theta_A+\theta_B)]^{m-1}\right\}\\&\left\{1-[1-\bar{h}_A(\theta_A+\theta_B)]\sum_{n_1=1}^{\infty}W_{n_1}(t)[\bar{h}_A(\theta_A+\theta_B)]^{n_1-1}\right\}\left\{1-[1-\bar{h}_B(\theta_A+\theta_B)]\sum_{n_2=1}^{\infty}V_{n_2}(t)[\bar{h}_B(\theta_A+\theta_B)]^{n_2-1}\right\}\end{aligned}$$

(8)

Using (5) in (8) we get

$$\begin{aligned}P(T>t)=&e^{-[\mu_1[1-\bar{g}_A(\theta_A)\bar{g}_B(\theta_A)]+\mu_{2A}[1-\bar{h}_A(\theta_A)]+\mu_{2B}[1-\bar{h}_B(\theta_A)]]t}+\\&e^{-[\mu_1[1-\bar{g}_A(\theta_B)\bar{g}_B(\theta_B)]+\mu_{2A}[1-\bar{h}_A(\theta_B)]+\mu_{2B}[1-\bar{h}_B(\theta_B)]]t}-\\&e^{-[\mu_1[1-\bar{g}_A(\theta_A+\theta_B)\bar{g}_B(\theta_A+\theta_B)]+\mu_{2A}[1-\bar{h}_A(\theta_A+\theta_B)]+\mu_{2B}[1-\bar{h}_B(\theta_A+\theta_B)]]t}\end{aligned}$$

(9)

It is known that

$$E(T^r)=r\int_0^{\infty}t^{r-1}P(T>t)dt,\quad r\geq 1 \qquad (10)$$

From(9)and (10) we get

$E(T)=\frac{1}{\tilde{A}}+\frac{1}{\tilde{B}}-\frac{1}{\tilde{C}}$

$V(T)=\frac{1}{\tilde{A}^2}+\frac{1}{\tilde{B}^2}-\frac{3}{\tilde{C}^2}-2(\frac{1}{\tilde{A}\tilde{B}}-\frac{1}{\tilde{B}\tilde{C}}-\frac{1}{\tilde{C}\tilde{A}})$

Hazard rate at time t $=\frac{\tilde{A}e^{-\tilde{A}t}+\tilde{B}e^{-\tilde{B}t}-\tilde{C}e^{-\tilde{C}t}}{e^{-\tilde{A}t}+e^{-\tilde{B}t}-e^{-\tilde{C}t}}$

Probability that recruitment takes place in (t, t+dt) given that there is no recruitment in [0,t]
$=P(t<T<t+dt/T>t)$
$=\frac{e^{-\tilde{A}t}(1-e^{-\tilde{A}t})+e^{-\tilde{B}t}(1-e^{-\tilde{B}t})-e^{-\tilde{C}t}(1-e^{-\tilde{C}t})}{e^{-\tilde{A}t}+e^{-\tilde{B}t}-e^{-\tilde{C}t}}$

Average residual time for recruitment given that there is no recruitment upto time t.

$$=E(T-t/T>t)=\frac{e^{-\tilde{A}t}/\tilde{A}+e^{-\tilde{B}t}/\tilde{B}-e^{-\tilde{C}t}/\tilde{C}}{e^{-\tilde{A}t}+e^{-\tilde{B}t}-e^{-\tilde{C}t}}$$

where

$$\tilde{A}=\mu_1[1-\bar{g}_A(\theta_A)\bar{g}_B(\theta_A)]+\mu_{2A}[1-\bar{h}_A(\theta_A)]+\mu_{2B}[1-\bar{h}_B(\theta_A)]$$

$$\tilde{B}=\mu_1[1-\bar{g}_A(\theta_B)\bar{g}_B(\theta_B)]+\mu_{2A}[1-\bar{h}_A(\theta_B)]+\mu_{2B}[1-\bar{h}_B(\theta_B)]$$

$$\tilde{C}=\mu_1[1-\bar{g}_A(\theta_A+\theta_B)\bar{g}_B(\theta_A+\theta_B)]+\mu_{2A}[1-\bar{h}_A(\theta_A+\theta_B)]+\mu_{2B}[1-\bar{h}_B(\theta_A+\theta_B)]$$

(11)

From (9) one can prove the following results using law of total probability for expectation.

Average number of policy and transfer decisions required to make recruitment at T
$=(\mu_1+\mu_{2A}+\mu_{2B})E(T)$

Average total loss of manpower due to $N_P(T)$ and $N_{Trans.}(T)$ decisions
$=\{\mu_1[E(X_{Ai})+E(X_{Bi})]+\mu_{2A}E(Y_{Aj})+\mu_{2B}E(Y_{Bj})\}E(T)$

Case (iii) let $Z=Z_A+Z_B$

In this case $k(z)=\frac{\theta_A\theta_B}{\theta_A-\theta_B}[e^{-\theta_B z}-e^{-\theta_A z}]$ (12)

From (1),(2)and (12)

$$P(T > t)$$
$$= \frac{\theta_A}{\theta_A - \theta_B} \sum_{m=0}^{\infty} [F_m(t) - F_{m+1}(t)] \, [\bar{g}_A(\theta_B)\bar{g}_B(\theta_B)]^m \sum_{n_1=0}^{\infty} [W_{n_1}(t) - W_{n_1+1}(t)] \, [\bar{h}_A(\theta_B)]^{n_1} \sum_{n_2=0}^{\infty} [V_{n_2}(t) - V_{n_2+1}(t)][\bar{h}_B(\theta_B)]^{n_2}$$
$$- \frac{\theta_B}{\theta_A - \theta_B} \sum_{m=0}^{\infty} [F_m(t) - F_{m+1}(t)] \, [\bar{g}_A(\theta_A)\bar{g}_B(\theta_A)]^m \sum_{n_1=0}^{\infty} [W_{n_1}(t) - W_{n_1+1}(t)] \, [\bar{h}_B(\theta_A)]^{n_1} \sum_{n_2=0}^{\infty} [V_{n_2}(t) - V_{n_2+1}(t)][\bar{h}_B(\theta_A)]^{n_2}$$

On simplification

$$P(T > t) = \frac{1}{\theta_A - \theta_B} \quad \{\theta_A e^{-\bar{B}t} - \theta_B e^{-\bar{A}t}\} \tag{13}$$

From (10) and (13) we get

$$E(T) = \frac{1}{\theta_A - \theta_B} \left\{ \frac{\theta_A}{\bar{B}} - \frac{\theta_B}{\bar{A}} \right\}$$

$$V(T) = \frac{2}{\theta_A - \theta_B} \left\{ \frac{\theta_A}{\bar{B}^2} - \frac{\theta_B}{\bar{A}^2} \right\} - \frac{1}{[\theta_A - \theta_B]^2} \left\{ \frac{\theta_A{}^2}{\bar{B}^2} + \frac{\theta_B{}^2}{\bar{A}^2} - \frac{2\theta_A\theta_B}{\bar{A}\bar{B}} \right\}$$

Hazard rate at time t $= \frac{\theta_A \bar{B} e^{-\bar{B}t} - \theta_B \bar{A} e^{-\bar{A}t}}{\theta_A e^{-\bar{B}t} - \theta_B e^{-\bar{A}t}}$

Probability that recruitment takes place in (t, t+dt) given that there is no recruitment in [0,t]

$$= P(t < T < t + dt / T > t)$$
$$= \frac{\theta_A e^{-\bar{B}t}(1 - e^{-\bar{B}dt}) - \theta_B e^{-\bar{A}t}(1 - e^{-\bar{A}dt})}{\theta_A e^{-\bar{B}t} - \theta_B e^{-\bar{A}t}}$$

Average residual time for recruitment given that there is no recruitment upto time t.

$$= E(T - t / T > t) = \frac{\theta_A e^{-\bar{B}t} / \bar{B} - \theta_B e^{-\bar{A}t} / \bar{A}}{\theta_A e^{-\bar{B}t} - \theta_B e^{-\bar{A}t}}$$

Expected number of policy and transfer decisions required to make recruitment at T

$$= (\mu_1 + \mu_{2A} + \mu_{2B}) E(T)$$

Mean total loss of manpower due to $N_P(T)$ and $N_{Trans.}(T)$ decisions

$$= \{\mu_1 [E(X_{Ai}) + E(X_{Bi})] + \mu_{2A} E(Y_{Aj}) + \mu_{2B} E(Y_{Bj})\} E(T)$$

Note:

If X_{Ai}, X_{Bi}, Y_{Aj} *and* Y_{Bj} follow exponential distribution with parameters $\alpha_{1A}, \alpha_{1B}, \alpha_{2A}$ and α_{2B} respectively, then $\bar{g}_A(\theta_A + \theta_B) = \frac{\alpha_{1A}}{\alpha_{1A} + \theta_A + \theta_B}$, $\bar{g}_B(\theta_A + \theta_B) = \frac{\alpha_{1B}}{\alpha_{1B} + \theta_A + \theta_B}$,

$$\bar{h}_A(\theta_A + \theta_B) = \frac{\alpha_{2A}}{\alpha_{2A} + \theta_A + \theta_B} \text{ and } \bar{h}_B(\theta_A + \theta_B) = \frac{\alpha_{2B}}{\alpha_{2B} + \theta_A + \theta_B}. \tag{14}$$

and explicit analytical expression for the performance measures obtained for the cases (i),(ii) and (iii) can be determined using (14).

Numerical Example:

The performance measures namely mean and variance of the time to recruitment are obtained by varying only one parameter and the results are tabulated. The impact of the nodal parameters α_{1A} and μ_1 on these performance measures is given as findings.

Table.1

(α_{1B}=0.1; α_{2A}=0.1; α_{2B}=0.2; μ_{2B}=0.01; μ_{2A}=0.05; θ_A=1.0; θ_B=0.1)

α_{1A}	μ_1	Case(i)		Case(ii)		Case(iii)	
		$E(T$	$V(T)$	$E(T)$	$V(T)$	$E(T)$	$V(T)$
0.1	0.05	9.62	92.54	15.24	231.16	15.80	105.21
0.3	0.05	9.67	93.52	16.84	281.99	17.56	135.59
0.5	0.05	9.70	94.27	17.45	302.73	18.23	147.91
0.7	0.05	9.73	94.85	17.78	313.98	18.59	154.52
0.9	0.05	9.76	95.33	17.98	321.03	18.80	158.62
0.05	0.1	6.49	42.20	8.97	80.39	9.22	33.08
0.05	0.3	2.83	8.01	3.59	12.93	3.67	4.92
0.05	0.5	1.81	3.27	2.25	5.05	2.29	1.88
0.05	0.7	1.33	1.76	1.63	2.67	1.66	0.98
0.05	0.9	1.05	1.10	1.28	1.65	1.31	0.60

Findings:

From the above table the following inference are presented which agree with reality,

- When α_{1A} increases and keeping all the other parameters fixed, the mean and variance of time to recruitment increase for all the three cases.
- When μ_1 increases and keeping all the other parameters fixed, the mean and variance of time to recruitment decrease for all the three cases.

3. Conclusion

The manpower planning model developed in this paper can be used to plan for the adequate provision of manpower for the organization at graduate, professional and management levels in the context of attrition. There is a scope for studying the applicability of the designed model using simulation. Further, by collecting relevant data, one can test the goodness of fit for the distributions assumed in this paper. The findings given in this paper enable one to estimate manpower gap in future, thereby facilitating the assessment of manpower profile in predicting future manpower development not only on industry but also in a wider domain.

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