

One Twenty Phase Code Design for Radar

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Abstract

Sequences with good autocorrelation Properties are useful for radar and communication applications. In this paper One Twenty Phase sequences are synthesized using Modified Genetic Algorithm (MGA). MGA is used as a statistical technique for obtaining approximate solutions to combinatorial optimization problems. This algorithm combines the good methodologies of the two algorithms like global minimum converging property of Genetic Algorithm (GA) and fast convergence rate of Hamming scan algorithm. The synthesized sequences have autocorrelation Properties better than well-known binary MPS code and Frank codes. The synthesized sequences also have complex signal structure which is difficult to detect and analyze by enemy electronics support measure.

Keywords

Ambiguity Function, Autocorrelation, Discrimination Factor, Hamming Scan, Polyphase Codes, Genetic Algorithm, Radar Signal.

1. Introduction

Sequences with low aperiodic autocorrelation sidelobe levels are useful for channel estimation, radar, and spread spectrum communication applications. Sequences achieving the minimum peak aperiodic autocorrelation sidelobe level one are called Barker Sequences. The aperiodic autocorrelation function (ACF) of sequence S of length N is given by,

$$A(k) = \begin{cases} \sum_{n=0}^{N-k-1} s_n s_{n+k}^* ; & 0 \leq k \leq N-1 \\ \sum_{n=0}^{N+k-1} s_n s_{n-k}^* ; & -N+1 \leq k \leq 0 \end{cases} \quad \dots(1)$$

If all the sidelobes of the ACF of any polyphase sequence are bounded by

$$|A(k)| \leq 1, \quad 1 \leq |k| \leq N-1 \quad \dots (2)$$

then the sequence is called a generalized Barker sequence or a polyphase Barker sequence. In 1953 [1] Barker introduced binary sequences for lengths $N = 2, 3, 4, 5, 7, 11$, and 13, fulfilling the condition in (2). The binary Barker can be regarded as a special case of polyphase Barker sequences. Binary codes that yield minimum peak sidelobes but do not meet the Barker condition are often called Minimum Peak Sidelobe (MPS) codes[2]. If the sequence elements are taken from an alphabet of size M, consisting of the M^{th} roots of unity.

$$S_m = \exp\left\{2\pi i \frac{m}{M}\right\} =: \exp(i\phi_m) \quad 0 \leq m \leq M-1 \quad \dots (3)$$

the sequence is alternatively named an M-phase Barker sequence. In 1965, Golomb and Scholtz [3] first investigated generalized Barker sequences and presented six phases Barker sequence of lengths $N \leq 13$. Recently, in [4] polyphase Barker sequences of lengths 46-63 were presented, wherein, an alphabet size of 2000 had to be used. However, polyphase Barker sequences for larger lengths require larger alphabets and the possibility for exhaustive search diminishes. The other well known polyphase codes with ideal autocorrelation are Frank codes [5]. Frank codes exist in perfect square length only. The synthesis of polyphase codes with good correlation properties is a nonlinear multivariable optimization problem, which is usually difficult to tackle. The Genetic Algorithm (GA) technique proved to be an efficient and powerful tool to find optimal or near optimal solutions for complex multivariable nonlinear functions but has slow convergence rate. The concept of Hamming scan algorithm has been employed for obtaining the pulse compression sequences at larger lengths with good correlation properties [5,6]. This algorithm has fast convergence rate but has demerit viz., the tendency to be stuck with local minima. The MGA has global minimum estimation capability of GA algorithm and fast convergence rate of Hamming scan algorithm [6,7,8]. Binary code is one of the most commonly used radar pulse compression signals due to the easy signal generation and processing [2,9,10]. Polyphase signal has larger main lobe-to-peak sidelobe ratio over binary signal of the same code length. In addition, polyphase waveforms have a more complicated signal structure and thus, are more difficult to detect and analyze by an enemy's electronic support measures (ESMs). With the maturity of digital signal processing, the generation and

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processing of polyphase signals has become easy and less costly. Therefore, polyphase code is increasingly becoming a favorable alternative to the traditional binary code for radar signals and can be used as the basic code for radar signal design. In this paper, MGA has been used for the design of One Twenty Phase sequences with good Discrimination factor.

2. One Twenty Phase Sequences

The One Twenty Phase sequence of length N bits is represented by a complex number sequence

$$\{s(n) = e^{j\phi_m(n)}, \quad n = 1, 2, \dots, N\} \quad \dots \quad (4)$$

Where $\phi_m(n)$ is the phase of nth bit in the sequence and lies between 0 and 2π . If the number of the distinct phases available to be chosen for each bit in a code sequence is M, the phase for the bit can only be selected from the following admissible values:

$$\phi_m(n) \in \left\{0, \frac{2\pi}{M}, 2\frac{2\pi}{M}, \dots, (M-1)\frac{2\pi}{M}\right\} \quad \dots \quad (5)$$

$$= \{\psi_1, \psi_2, \dots, \psi_M\}$$

For example if $M = 4$, then values of $\{\psi_1, \psi_2, \psi_3 \text{ and } \psi_4\}$ will be $0, \pi/2, \pi$ and $3\pi/2$ respectively.

Considering a One Twenty Phase sequence S with code length N, one can concisely represent the phase values of S with the following 1 by N phase matrix:

$$S = [\phi_m(1), \phi_m(2), \phi_m(3), \dots, \phi_m(N)] \quad \dots \quad (6)$$

where all the elements in the matrix can only be chosen from the phase set in (5).

A more practical approach to design One Twenty Phase sequences with properties in (2) is to numerically search the best One Twenty phase sequences by minimizing a cost function that measures the degree to which a specific result meets the design requirements. For the design of One Twenty Phase sequences used in radar and communication the cost function is based on the sum of autocorrelation side lobe peaks. Hence, from (1) the cost function can be written as,

$$E = \sum_{k=1}^{N-1} |A(k)| \quad \dots \quad (7)$$

The minimization of cost function in (7) generates a One Twenty Phase sequences that are automatically constrained by (2). In this optimization we have minimize the autocorrelation sidelobe.

3. Discriminating Factor (DF)

The discriminating factor (DF) as defined by Golay is ratio of mainlobe peak value to the magnitude of sidelobe peak value of Autocorrelations function of sequence S. The DF, mathematically is defined as follows[11].

$$DF = \frac{A(0)}{\max_{k \neq 0} |A(k)|} \quad \dots \quad (8)$$

The denominator is a measure of the peak sidelobe value and is related to the L_∞ norm of the sidelobes.

4. Modified Genetic Algorithm (MGA)

Modified Genetic Algorithm is proposed as a statistical technique for obtaining approximate solutions to combinatorial optimization problems. The proposed algorithm is a combination of Genetic Algorithm (GA) and Hamming Scan algorithms. It combines the good methodologies of the two algorithms like global minimum converging property of GA algorithm and fast convergence rate of Hamming scan algorithm. The demerit of Hamming scan algorithm is that it gets stuck in the local minimum point because it has no way to distinguish between local minimum point and a global minimum point. Hence it is sub-optimal. The drawback in Genetic algorithm is that it has a slow convergence rate because even though it may get closer to the global minimum point, it may skip it because of the methodology it employs. The MGA overcomes these drawbacks. It is quite effective to combine GA with Hamming Scan (HSA) Algorithm. GA tends to be quite good at finding generally good global solutions, but quite inefficient at finding the last few mutations to find the absolute optimum. Hamming Scan are quite efficient at finding absolute optimum in a limited region. Alternating MGA improve the efficiency of GA while overcoming the lack of robustness of HSA. MGA are introduced as a computational analogy of adaptive systems. They are modeled loosely on the principles of the evolution via natural selection, employing a population of individuals that undergo selection in the presence of variation-inducing operators such as mutation and recombination. A fitness function is used to evaluate individuals, and reproductive success varies with fitness.

5. One Twenty phase Sequences Design Using MGA

The flowchart of MGA for optimizing the One Twenty phase codes is shown in Figure1. In this flow chart ; g,

d, DF, L, N, i, p_c , p_m mean number of desired iteration, desired number of optimum sequences, desired Discriminating Factor, number of sequences in set, sequence length, iteration counter, probability of crossover and probability of mutation. The computational cost for searching the best One Twenty phase, sequence of length N, through an exhaustive search, i.e., minimizing (7), is of the order of $(120)^N$ grows exponentially with the code length. Therefore, the numerical optimization of One Twenty phase sequences is an NP-complete problem. During the optimization process of One Twenty phase sequence sets, the random search is carried out through "crossover" and "mutation", i.e., randomly selecting an entry in the (6) and replacing it with different admissible value with probability of P_c and p_m respectively. The next step of the algorithm is to invoke the Hamming scan to find the optimum sequence in the vicinity of sequence selected by GA.

6. Ambiguity Function

The radar signal design is actually based on the ambiguity function rather than autocorrelation function. The ambiguity function of transmit waveform specifies the ability of the sensor to resolve targets as a function of delay (τ) and Doppler (ν). The ideal transmit signal would produce an ambiguity function with zero value for all non-zero delay and Doppler (i.e., a "thumbtack"), indicating that the responses from dissimilar targets are perfectly uncorrelated. It is well known that if the ambiguity function is sharply peaked about the origin, then simultaneous range and velocity resolution capability is good.

Ambiguity function $|\chi(\tau, \nu)|$ can be defined as[2]

$$|\chi(\tau, \nu)| = \left| \int_{-\infty}^{\infty} u(t) u^*(t - \tau) \exp(j2\pi\nu t) dt \right| \quad \dots (9)$$

where $u(t)$ is the transmitted signal.

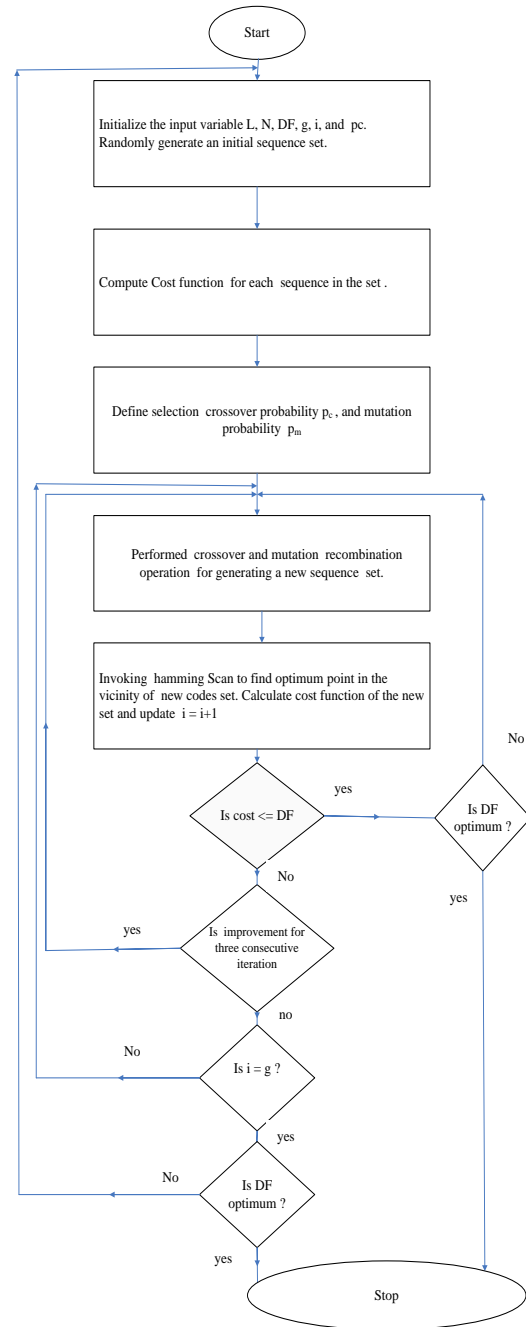


Figure 1: Flow chart of Modified Genetic Algorithm

Ambiguity Function has been used to assess the properties of the transmitted waveform as regards to its target resolution, measurement accuracy, ambiguity, and response to clutter and effect of Doppler [2,10,12-16]. Unlike the linear frequency modulation (LFM) signals[2], the numerically designed Ninety phase sequences have thumbtack ambiguity diagram, and thus, the matched filtering results are very sensitive to the

Doppler frequency (ν) in the radar echoes due to target movement. It can be seen that the output signal amplitude is not significantly reduced (signal loss < 3 dB) if the Doppler frequency is less than $0.5/T$, i.e.

$$|\nu|T < 0.5 \quad \dots \quad (10)$$

where T is the signal time duration equal to Nt_b , where t_b is the duration of sub pulse. Therefore, if (10) is satisfied, the Doppler effect on the processing result is negligible; otherwise, the correction processing must be conducted. A simple way to minimize the Doppler effect is to select the signal time duration such that (10) is satisfied for all expected target speeds. Another approach for overcoming the Doppler effect is to use a bank of Doppler-matched filters for every signal. Each of the Doppler-matched filters is designed to match a different Doppler-shifted version of the signal. Target detection is based on the maximum output from the Doppler-matched filter bank. The Doppler shift frequencies and the number of the matched filters are chosen such that the signal loss is limited to a tolerable level (such as 3 dB) for all possible target speeds.

7. Results

One Twenty Phase sequences are designed using the MGA, the length of the sequence, N , is varied from 6 to 400. The cost function for the optimization is based on (7). In this paper all, the DF values are obtained using Pentium - IV, processor. Table 1 shows the Comparison of Discrimination Factors of One twenty phase synthesized sequences with Binary MPS codes. In table 1, column 1 shows sequence length, N , column 2 shows DF of binary MPS codes and column 3 shows the DF of One Twenty Phase sequences. Table 2 shows the comparison of Discrimination Factors of One twenty phase synthesized sequences with Frank codes. In table 2, column 1 shows sequence length, N , column 2 shows DF of Frank codes and column 3 shows the DF of One Twenty Phase sequences. The figs (2) and (3) are the graphical representation of DF comparison of Minimum Peak Sidelobe (MPS) Binary codes and Frank codes with synthesized One twenty-phase sequences whose values are shown in table I and II respectively. As shown in figure (2) and (3) the DF of One twenty-phase sequences are far better than both MPS binary and well known Frank codes. Figure (4) shows the signal structure (amplitude and phase) of One twenty phase synthesized sequence of length ($N = 400$). Figure (5) shows the autocorrelation function of One twenty phase synthesized sequence and MPS Binary code of length ($N = 44$). As shown in the figure (5) the peak sidelobe label of One twenty phase sequence is 10 dB less than

MPS Binary code. Figure (6) shows ambiguity diagram of One twenty phase synthesized sequence of length ($N = 400$). Figure (7) shows zoomed (in time and Doppler) version of figure (6). As shown in the figures the ambiguity diagram is Thumbtack. Figure (8) shows the effect of Doppler shift on autocorrelation function of One Twenty phase synthesized sequence of length $N = 400$.

From results analysis it can be shown that One Twenty phase sequences have DF better than MPS binary codes and well known Frank codes. Apart from having better DF the synthesized sequences have a more complicated signal structure than Frank and binary codes, and thus, are more difficult to detect and analyze by an enemy's electronic support measures (ESMs).

Table 1: Comparison of Discrimination Factor of One twenty phase synthesized sequences with Binary MPS codes

Sequence Length (1)	Binary MPS codes (2)	One Twenty phase sequences (3)
6	3.0	6
25	12.5	25
36	12.0	29.1
41	12.7	41
42	14.0	42
43	14.3	43
44	14.7	44

Table 2: Comparison of Discrimination Factor of One Twenty Phase synthesized sequences with Frank codes

Sequence Length(1)	Frank codes(2)	One Twenty phase Sequences (3)
16	11.3	16
25	15.5	25
36	18.0	29.1
49	21.8	27.8
64	24.4	40.9
81	28.1	37.2
100	30.9	41.4
121	34.4	44.6
225	47.0	63.7
361	59.66	67.2
400	62.57	69.3

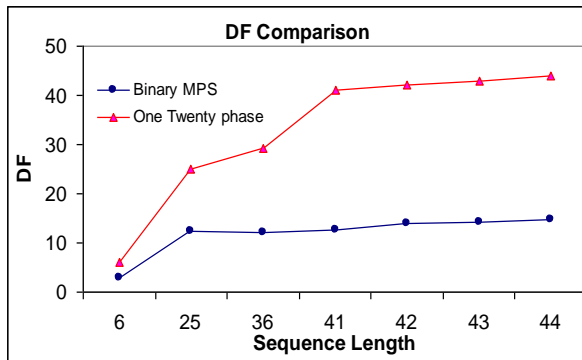


Figure 2: Comparison of Discrimination Factor of Minimum peak Sidelobe Binary codes and One twenty Phase sequences

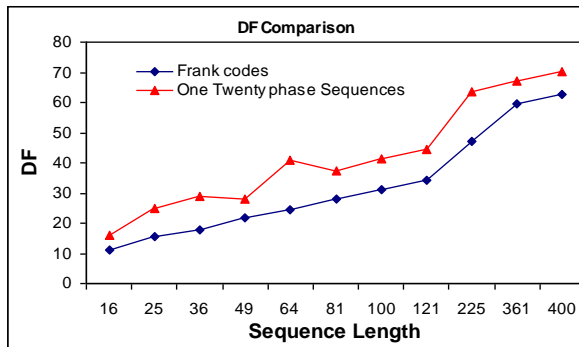


Figure 3: Comparison of Discrimination Factor of Frank codes and One Twenty Phase sequences

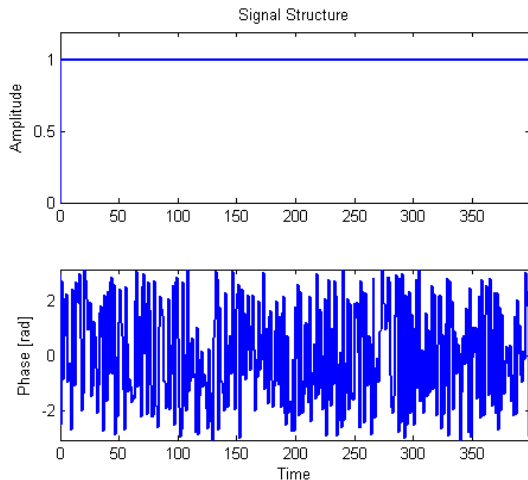


Figure 4: Signal structure of One twenty phase synthesized sequence of length (N = 400).

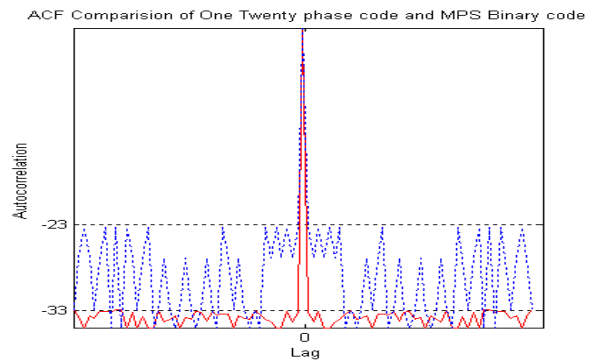


Figure 5: Autocorrelation function Comparison of One twenty phase synthesized sequence and MPS Binary code of length N = 44

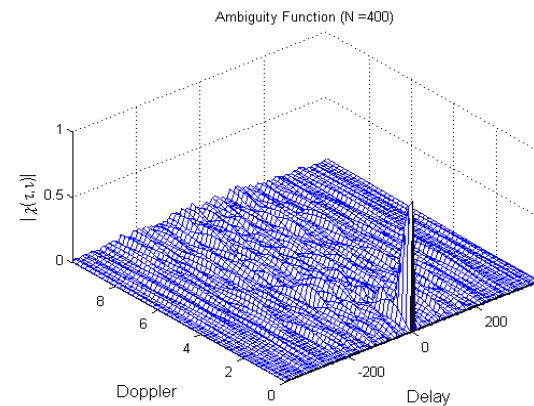


Figure 6: Ambiguity diagram of One Twenty phase synthesized sequence of length N = 400

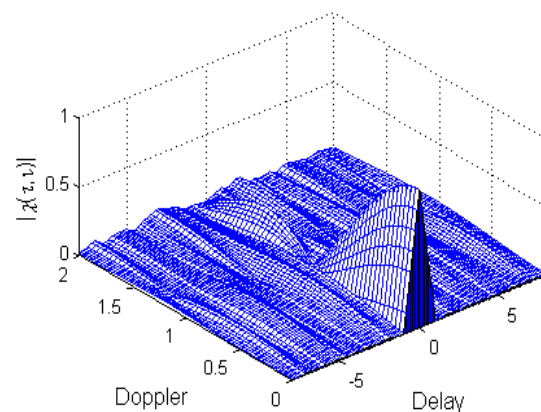


Figure 7: Ambiguity diagram zoomed of fig(5) in time and Doppler

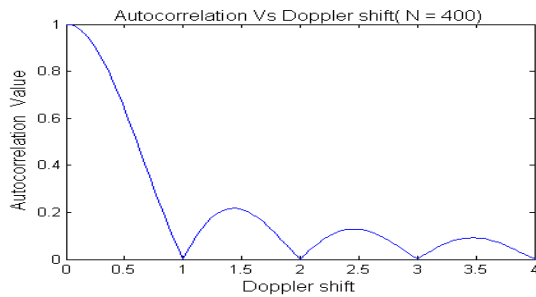


Figure 8: Effect of Doppler shift of Autocorrelation function of One twenty phase synthesized sequence of length (N = 400)

8. Conclusions

An effective Modified Genetic algorithm has been used for designing the One Twenty phase coded sequences with good autocorrelation properties. The synthesized sequences can be used in radar systems and spread spectrum communications for significantly improving performance of the system. The One twenty phase sequences are designed up to a length of 400. The synthesized results presented in this paper not only have better correlation properties but also have more complicated signal structure which is difficult to detect and analyze by an enemy's electronic support measures (ESMs)[17]. Hence, it can be concluded that the design results are very useful for radar as well as spread spectrum communication systems.

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