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# Robust LQR Control Design of Gyroscope

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## Abstract

***The basic problem in designing control systems is the ability to achieve good performance in the presence of uncertainties such as output disturbances, measurement noise or unmodeled dynamics (i.e. robust controllers). Recent development in the area has been directed towards developing a consistent design methodology within this uncertain environment. The attitude control/ momentum management of the space station poses a typical problem in a highly uncertain environment (such as mass properties of the Space Station and environmental disturbances as well as parametric uncertainties). The objective of this research is to use LQR control for the position control of spin axis rotor position at reference value in the presence of parametric uncertainties, external unmeasurable disturbances and system inherent non-linearity with different type's reference tracking signal are considered extensively in this paper.***



## Keywords

***Gyroscope Control; LQR Control; Optimal Control; I-LQR Control; Reference tracking scheme; Robust Control.***


## 1. Introduction

A spacecraft can use either internal or external actuators to control its attitude. Generally external actuators such as thrusters are used for large fast slewing maneuvers. However thrusters are not ideal for precision attitude control due to their discontinuous nature, due to this reason such kind control strategy is not implementable in many situations. Internal actuators can be momentum exchange devices, such as momentum wheels and control moment gyros, or non-moving devices like magnetic torques. Momentum wheels can perform precise maneuvers and maintain attitude; however the varying wheel speeds tend to excite structural dynamics.

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Momentum wheels [1,2] require torques proportionate to the desired output torque which makes them non-ideal for rapid slewing. A control moment gyro has a flywheel mounted on a motor/actuator that spins at a constant relative speed at all time of instant. The space station is employed with CMG (control moment gyroscope) as a primary actuating device during normal flight mode operation. The objective of the CMG flight control system is to hold the space station at a fixed attitude relative to the LVLH frame. In the presence of continuous environmental disturbances [3-5] CMGs will absorb momentum in an attempt to maintain the Space Station at a desired attitude. The CMGs will eventually saturate, resulting in loss of effectiveness of the CMG system as a control effector element.

Linear Quadratic Regulator (LQR) [6-13] design technique is well known in modern optimal control theory and has been widely used in many applications. Designed controller has a very nice robustness property, [14] i.e., if the process is of single-input and single-output, then the control system has at least the phase margin of 60 and the gain margin of infinity. This attractive property appeals to the practicing engineers. Thus, the LQR theory has received considerable attention after 1950s. In the context of optimal PID tuning, typical performance indices are the integral of squared error and time weighted error. With this kind of performance criterions, the integral of squared error (squared time weighted error) is calculated using Astrom's integral algorithm recursively if the process transfer function is known [15].

In optimal control one attempt to find a controller that provides the best possible performance with respect to some given measure of performance. E.g., the controller that uses the least amount of control-signal energy to take the output to zero. In this case the measure of performance (also called the optimality criterion) would be the control-signal energy to the plant. In general, optimality with respect to some criterion is not the only desirable property for a controller, but also help to reject disturbances from input channel. One would also like stability of the closed-loop system, good gain and phase margins, robustness with respect to unmodeled dynamics etc.

Major contributions of this paper are as follow: In order to achieve optimal performance of gyroscope model, initially LQR control is designed for state feedback gain matrix. Mathematical modeling of gyroscope is presented. Later on Integral-LQR control is designed to achieve robust performance in the presence parametric uncertainties and inherent non-linearity of system with different reference signals like sinusoidal, square and sawtooth wave to test the system performance. The entire designed control algorithms are tested on gyroscope plant.

Rest of the paper is organized as follows: Section 2 describes the problem formulation and LQR control design concept. The mathematical model of Gyroscope is explained in Section 3. Simulation results and discussions are presented in Section 4 and the paper concludes in Section 5.

## 2. Problem Formulation and LQR Control Design

Consider a Single Input Single Output (SISO) linear non-time invariant system as

$$\dot{x}(t) = Ax(t) + Bu(t) \qquad (1)$$
$$y(t) = Cx(t) + Du(t)$$

Where *A*, *B* are system parameter matrices. *C* is output matrix. *x(t)* is the internal state vector of system. *y(t)* is the output of the system, which is interest of control. *u(t)* is the control input.

Generalized control scheme is shown in fig. (1). For designing of controller internal state feedback is required. Based on the states information controller is designed.
For the derivation of the linear quadratic regulator, we assume the plant to be written in state-space form $\dot{x} = Ax + Bu$, and that all of the *n* states *x* are available for the controller. The feedback gain is matrix *K*, implemented as $u(t) = -K(x(t) - x_d(t))$. The system dynamics are then written as

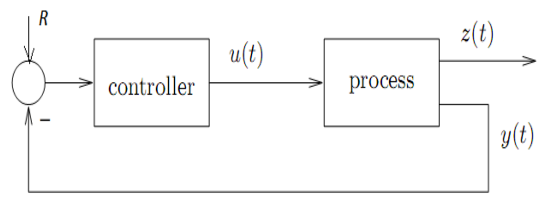


**Figure 1: Generalized block diagram of LQR control**

The control objective is to use state feedback to stabilize the system and prevent flutter. We focus on the infinite time Linear-Quadratic Regulator problem. This approach leads to a full state-feedback controller of the form $u = -K_c x$ to maintain stability. We show that a stable solution exists and use the algebraic Riccati Equation to solve for an optimal control $u^*$. MATLAB/ Simulink software is used in all numerical simulations to test plant with different conditions.

We present a general procedure for solving optimal control problems, using the calculus of variations.

$$J = \int_0^\infty [x(t)^T Qx(t) + u(t)^T Ru(t)]dt \qquad (2)$$

*Q* is an *n X n* symmetric positive semi definite matrix and *R* is an *m X m* symmetric positive definite matrix. The matrix *Q* can be written as $Q = M^T M$, where *M* is a *p X n* matrix, with $p < n$. With this representation

$$x^T Qx = x^T M^T Mx = z^T z \qquad (3)$$

where $z = Mx$ can be viewed as a controlled output.

**Assumptions:**

1. Pair *(A, B)* is Controllable.
2. Pair *(Q,R)* are the controller design parameters. Large Q penalizes transients of *x*, large *R* penalizes usage of control action *u(t)*.

## 3. Modelling of Gyroscope Plant

The following pair of equations [17] describes the motion of the Gyroscope plant and where dynamics can be obtained by

$$J_y \ddot{\theta} + h\dot{\varphi} = M_y \qquad (3)$$

$$-h\dot{\theta} + J_z \ddot{\varphi} = 0 \qquad (4)$$

Using eq. (3) and (4), deriving the open loop transfer function *G(s)* for the gyroscope plant from Laplace transform of z-axis position $\varphi$ as output of plant to Laplace transform of control torque input $M_y$.

Taking Laplace transform of equations (3) and (4), one can find the transfer function as

$$J_y\theta s^2 + h\varphi s = M_y \quad (5)$$

$$-h\theta s + J_y\varphi s^2 = 0 \quad (6)$$

Re-arranging eq. (5) and (6) to solve for $\theta$ we get:

$$\theta = \frac{J_z\varphi s}{h} \quad (7)$$

Substituting (5) back into (3) and rearranging, one can find the transfer function after some simplification as

$$\frac{\varphi}{M_y} = \frac{h}{J_y J_z s^3 + h^2 s} \quad (8)$$

which is the open loop transfer function G(s) for the gyroscope plant.

Now converting nominal transfer function of gyroscope present into eq. (8) into state space form as presented into (1),

Using equation (3) and (4) and the state vector given below, obtain the complete state-space representation for the gyroscope plant. Provide both parametric and numerical values for A and B matrices. Internal state vector of plant is selected as,

$$x = \begin{pmatrix} \theta \\ \varphi \\ \dot{\varphi} \end{pmatrix} \quad (9)$$

Re-arranging (3), one can write as,

$$\ddot{\theta} = \frac{-h}{J_y}\dot{\varphi} + \frac{1}{J_y}M_y \quad (10)$$

Rearranging (3) results in,

$$\dot{\varphi} = \frac{h}{J_z}\theta \quad (11)$$

Combining (9), (10) and (8), one can write dynamics of gyroscope plant as

$$x = \begin{pmatrix} \theta \\ \varphi \\ \dot{\varphi} \end{pmatrix} = \begin{pmatrix} 0 & 0 & \frac{-h}{J_y} \\ 0 & 0 & 1 \\ \frac{h}{J_z} & 0 & 0 \end{pmatrix} \begin{pmatrix} \theta \\ \varphi \\ \dot{\varphi} \end{pmatrix} + \begin{pmatrix} \frac{1}{J_y} \\ 0 \\ 0 \end{pmatrix} M_y \quad (11)$$

The output of the gyroscope plant is the variable $\varphi$, the angle about the z-axis. Therefore the second equation given in (7) takes the following form,

$$y = \varphi = [0 \quad 1 \quad 0]\begin{pmatrix} \theta \\ \varphi \\ \dot{\varphi} \end{pmatrix} \quad (12)$$

Substituting the parameter values from table-I, one can work on these equations as,

$$A = \begin{pmatrix} 0 & 0 & \frac{-h}{J_y} \\ 0 & 0 & 1 \\ \frac{h}{J_z} & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & -418.88 \\ 0 & 0 & 1 \\ 31.84 & 0 & 0 \end{pmatrix} \quad (13)$$

$$B = \begin{pmatrix} \frac{1}{J_y} \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 384.62 \\ 0 \\ 0 \end{pmatrix} \quad (14)$$

**Table 1: Nominal value of Gyroscope plant for simulation [16]**

| S.No | Parameter | Unit | Numerical Value |
|---|---|---|---|
| 1 | *h* | $Kg.m^2/s$ | 1.0891 |
| 2 | *Jy* | $Kg.m^2$ | 0.0026 |
| 3 | *Jz* | $Kg.m^2$ | 0.0342 |

## 4. Simulation results and discussion

In order to testing of designed control algorithm we have considered Quanser gyroscope plant for simulation example. Initially regulatory system is considered for initial results, where system states go to 0 from some initial arbitrary conditions. Plant states and control input is shown in Fig 2.

For tracking of filtered square wave we have considered same conditions as the previous case, results of square wave reference tracking signal shown in Fig 3.

For rest of the cases, 20% uncertainty is considered in system parameter matrices in order to test the robustness of controller. Fig 4 shows the plant state response for tracking of sinusoidal signal. Fig. 5 shows the system performance in the presence of parametric uncertainty to track sawtooth signal as reference.

Result shows that designed controller perform very well in the presence or parametric uncertainties and tracking of reference signals, where rate is not zero in steady states.

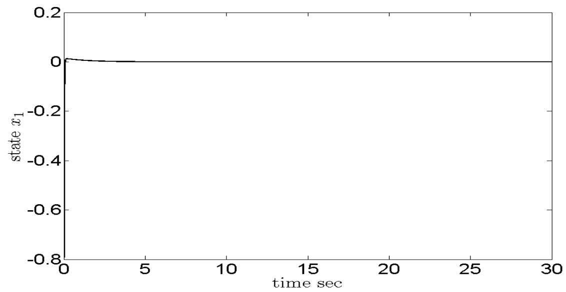


**Figure 2a: Plant state $x_1$**

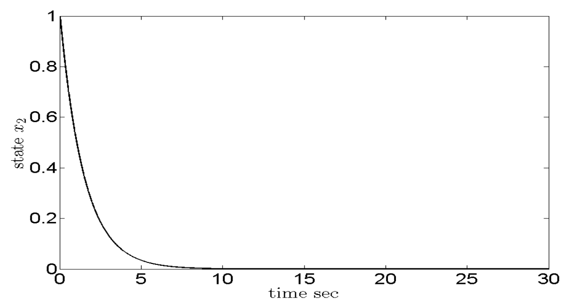


**Figure 2b: Plant state $x_2$**

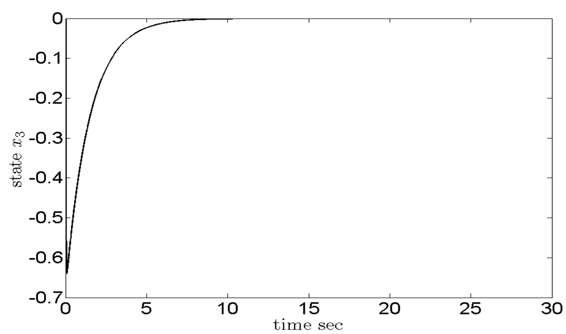


**Figure 2c: Plant state $x_3$**

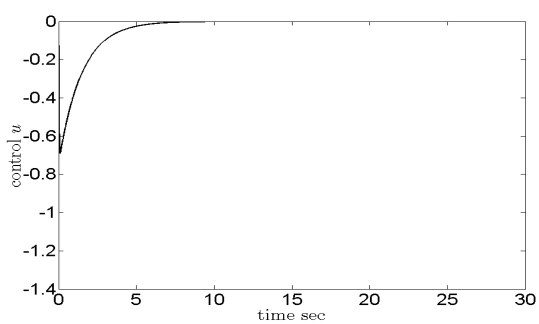


**Figure 2d: Control input $u$**

**Figure 2: Regulatory system performance**

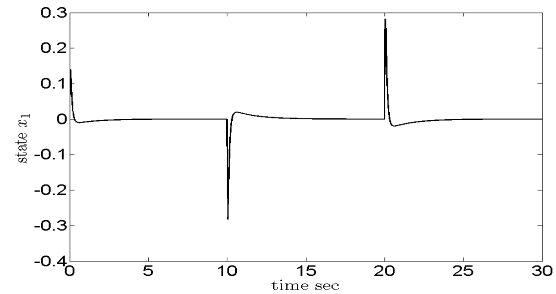


**Figure 3a: Plant state $x_1$**

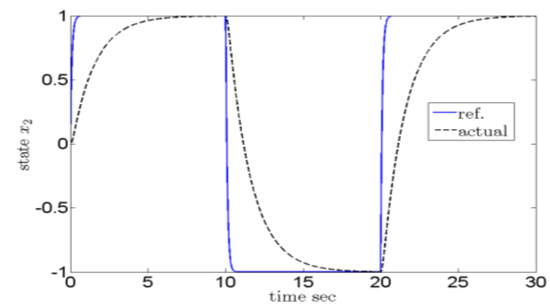


**Figure 3b: Plant state $x_2$**

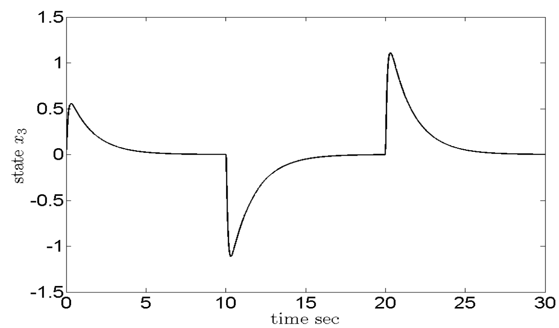


**Figure 3c: Plant state $x_3$**

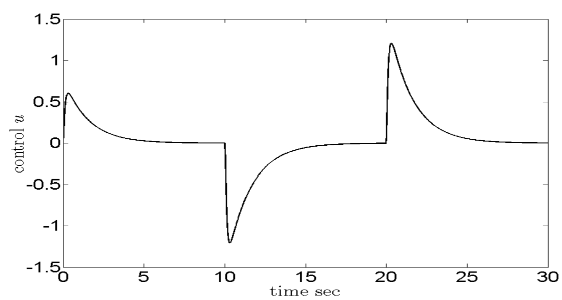


**Figure 3d: Control input $u$**

**Figure 3: System performance during square reference signal tracking with LQR control**

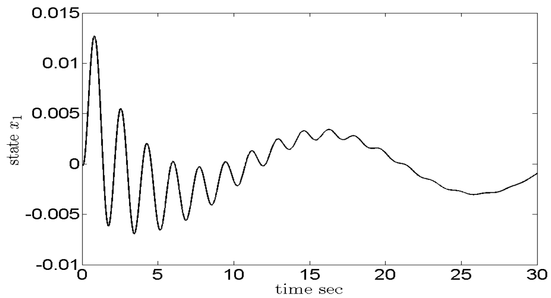


**Figure 4a: Plant state $x_1$**

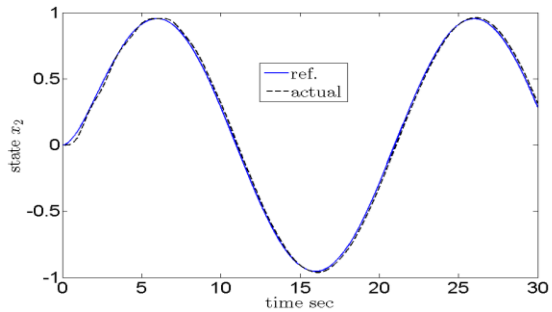


**Figure 4b: Plant state $x_2$**

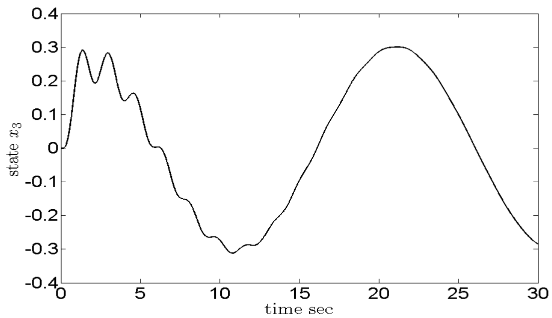


**Figure 4c: Plant state $x_3$**

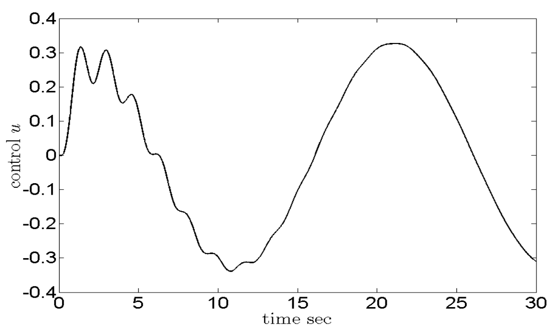


**Figure 4d: Control input $u$**

**Figure 4: System performance during sinusoidal reference signal tracking with I-LQR control**

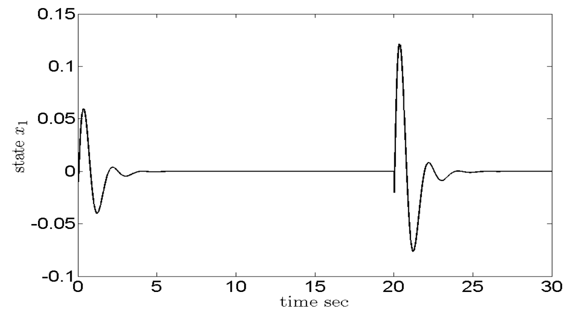


**Figure 5a: Plant state $x_1$**

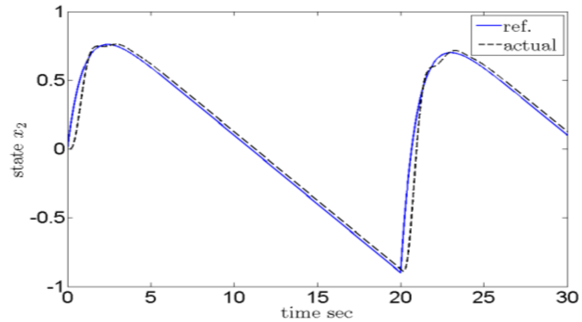


**Figure 5b: Plant state $x_2$**

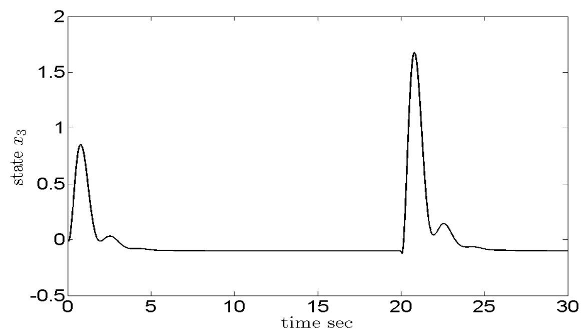


**Figure 5c: Plant state $x_3$**

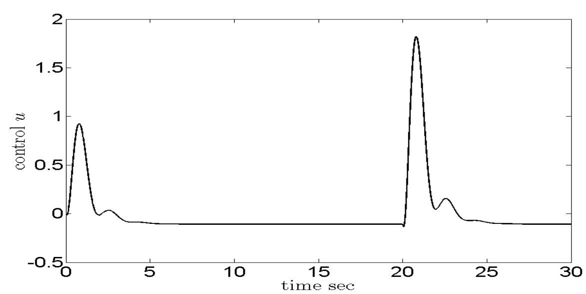


**Figure 5d: Control input $u$**

**Figure 5: System performance during sawtooth reference signal tracking with I-LQR control**

## 5. Conclusion

Design of LQR control is presented for linear time invariant SISO system. Systematic modeling of gyroscope based on equation of motion is presented in this paper. In order to improve system performance Integral term is added to remove the steady state error and tracking of sinusoidal, sawtooth signal in the presence of parametric uncertainties and external measurable disturbance. Designed controller scheme is applied to quanser gyroscope model with different tracking situation.

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