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An Efficient Image Denoising Method based on Fourth-Order Partial Differential Equations

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Abstract

Reduction of noise is essential especially in the field of image processing. Several researchers are continuously working in this direction and provide some good insights, but still there are lot of scope in this field. Noise mixed with image is harmful for image processing. In this paper we proposed an efficient PDE approach for reducing noise and blur parameters. In our approach we provide the comparison considering the image of leena and cameraman and improve the SNR ratio.

Keywords

Image Denoising, PDE, Noise reduction, SNR

1. Introduction

The active research in image processing is noise. If we think about the corrupted images, then we analyses that it is corrupted by random variations in intensity values which is the noise. It is because of the data acquisition process. The main aim of image denoising methods is to recover the original image or fetching the better quality image after reduction from a noisy one, in order to perform, in an easier and with a more semantic way to a task which is the part of image processing as image segmentation.

In this context several researches apply their work in this direction. Adaptive Directional Lifting (ADL) is one of the image compressions due to the characteristics of representing the edges and textures in images efficiently [1, 2]. Several researches have shown that the application of image denoising can also benefit from this technique [3, 4]. Because of this, it can effectively decorrelate the dependencies found over image discontinuities and compact high frequency components induced by image features into the lower level or low band pass. If we think about the Wavelet transform then it can be effectively capture singular points up to two dimensions means including one dimension, but it is fail in representing the major features like edge, color ,contour and so on. There are several directional and non-directional redundant transforms which are explored in different research papers, including the curvelet, contourlet, wedgelet, bandlet, and the steerable wavelet [5-8].

There are several approaches which are basically concern with denoise an image data, such as averaging filter, Median filter, Gaussian filter and Partial Differential Equations (PDE) approach. If we analyze the properties of good images then it will be with the less noise and minimize the blur or blur reduction is the important factor. The PDE approach is much effective and applies in several research like [9],[10]. But it is more effective if we apply fourth order partial differential equation. Applications of the PDE models can be widely found in a broad range of image restoration tasks such as denoising and enhancement [11] color image processing [12][13] and resolution. This provide us the future insight or work with the forth order partial differential equation with the same order in the direction of blur reduction.

We provide here an overview of Image denoising Technique. Other sections are arranged in the following manner: Section 2 introduces Image denoising; Section 3 describes about Recent Scenario; section 4 shows the proposed work. Section 5 shows the result analysis; Section 6 describes Conclusion.

2. Image Denoising

Image Denoising play an important role in Image processing task [14]. Remove the noise when the edges are in the preserving state is called image denoising. In the image processing task it is a major and most common problem. If we want a very high quality resolution images as the outcome then we must consider the noise parameters for reducing those parameters to achieve better. The main purpose or the aim of image denoising is to recover the main image from the noisy image [15].

$$V(i)=U(i) + N(i)$$

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Where v(i)is the observed value, u(i)is the “true” value and n(i) is the noise perturbation at a pixel i. If we analyses then there are several ways of model the noise. In some of the cases the noise is considerable. For modelling and framework purposes it is correct to additive white Gaussian Noise (AWGN) which is adaptive in nature to model the noise parameters. For that we also consider blur as the degrading performance categorization.

3. Literature Review

In 2012, Meenal et al. [14] survey and analysed different traditional image denoising method using different methods. They also suggest a new approach which provides a heterogeneous way of the above challenging issue. Their approach is the combination of three different approaches first is for blur, second is for noise and finally for blur and noise. After analysing several research works they analyse that not a single method can provide better method for blur and noise both. So their proposed solution can provides betterment in this issue.

In 2012, Meenal et al. [15] proposed an image denoising method using partial differential equation. In their proposed approach they proposed three different approaches first is for blur, second is for noise and finally for blur and noise. These approaches are compared by Average absolute difference, signal to noise ratio (SNR), peak signal to noise ratio (PSNR), Image Fidelity and Mean square error. They achieve result on different scenario. They also compare our result on the basis of the above five parameters and the result is better in comparison to the traditional technique.

In 2010, Saeid Fazli et al. [16] presents a new approach for image denoising based on Partial Differential Equations (PDE) using Artificial Intelligence (AI) techniques. The Nonlinear Diffusion techniques and PDE-based variation models are very popular in image restoring and processing but in this proposed heuristic method, Particle Swarm Optimization (PSO) is used for Complex PDE parameter tuning by minimizing the Structural SIMilarity (SSIM) measure. Complex diffusion is a generalization of diffusion and free Schrodinger equations which has properties of both forward and inverse diffusion. The proposed method by the author is confirmed by obtained simulation results of standard images.

In 2011, Changsheng Lang et al. [17] propose a combined transform image denoising algorithm based on morphological component analysis (MCA). Author suggests that the MCA method is used to separate the image into natural scene and linear singular structure. Curvelet transform threshold denosing is used in linear singular structure while wavelet transform deals with smooth part. This algorithm makes full use of respective advantages of the wavelet transform and curvelet transform. Experiment results show that the algorithm can better maintain the details characteristics in dealing with the image with linear singularity, and it has a better denosing performance for image than a simple wavelet thresholding or curvelet thresholding.

In 2012, Kehua Su et al. [18] introduce a sparse and redundant representations algorithm based on over complete learned dictionary to process different types of images. They use the K-SVD denoising framework and modify its initial dictionary, and then mainly focus on using it to study its denoising performance and suitability for different types of Images, and then compare it with some other image denoising algorithms. As to the remote sensing images denoising, the experiment results show that the K-SVD algorithm can leads to the state-of-art denoising performance at low noisy levels, but for high noisy levels, its performance isn’t good on PSNR and visual effect, that is it cannot retain the local details of images.

In 2012, Guo-Duo Zhang et al. [19] proposes an image denoising method based on support vector regression; also this paper describes several other methods of image denoising. Simulation results show that the method can save the image detail better, restore the original image and remove noise.

In 2011, Jia Liu et al. [20] proposed an image denoising method using partial differential equation and bi-dimensional empirical mode decomposition. The bi-dimensional empirical mode decomposition transforms the image into intrinsic mode function and residue. Different components of the intrinsic mode functions present different frequency of the image. The different with the classic method of partial differential equation denoising is that we use partial differential equation of the intrinsic mode functions to filter noise. Finally, they reconstruct the image with the filtered intrinsic mode functions and residue.

4. Proposed Work

After studying different approaches we observe that we can work with the reduction of noise and SNR. There is also some scope for the reducing the time of denoising while unaffected the accuracy. So in this section we using fourth order PDE for improving the SNR ratio.

The Fast Fourier Transform (FFT) method is an analytical technique for numerical analysis. It provide the approximate analysis to partial differential equations(PDE) which is generally calculated by the expansion in terms of function , which is also called basis method and used for calculating the unknown coefficients. Then we can apply the FFT method to partial differential equations like

$$\frac{\partial u}{\partial t} = c^2\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) + E(x, y)$$

This reduces the number of spatial variables until only a two-point boundary-value problem or initial-value problem remains, which is solved by standard methods. FFT is used for separate the variables so that it is used in the elaboration of partial differential equation. The PDE of Forth order is also help for reduction and separation because of the higher degree. It is much more flexible in the context of uses and separations of variables, so the identification is also easy and detectable and reduces the noise coefficients. The same separation is used for the boundary conditions.

But with the combination of FFT it is only used or applicable with the determined values or finite values. Let $\Theta = \Theta(\mathbf{r}, t)$ be the field variable (e.g., temperature or concentration), and let L be a differential operator which contains one or more spatial derivatives and perhaps also a time derivative. Many Partial differential equations can be represented in the below form:

$$L\Theta = S(\mathbf{r})$$

Where $S(\mathbf{r})$ is a function which is specified for dimension position. The differential operator is said to be linear if

$$L(a_1\Theta_1 + a_2\Theta_2) = a_1L\Theta_1 + a_2L\Theta_2$$

Where a_1 and a_2 are any constants and $\Theta_1(\mathbf{r}, t)$ and $\Theta_2(\mathbf{r}, t)$ are any functions [not necessarily solutions of $L\Theta = S(\mathbf{r})$]. We can also concern with the boundary values which must satisfy the boundary conditions which is alike in terms of that sepearted values in the same context to analyse and can serve as the same phenomena.

Consider the following PDE

$$\frac{\partial^2\Theta}{\partial x^2} + \frac{\partial^2\Theta}{\partial y^2} = 0$$

with the boundary conditions

$$\Theta(0, y) = 0, \Theta(1, y) = 0$$

$\Theta(x, 0) = f_0(x)$, $\Theta(x, 1) = f_1(x)$

The solution can be written in a series expansion as

$$\Theta(x, y) = \sum_{n=1}^{\infty} C_n(y)\Phi_n(x) \quad ,$$

In this equation $\Phi_n(x)$ are given by: $\Phi_n(x) = \sqrt{2}\sin(n\pi x), n = 1, 2, \ldots$

We follow the following steps:

```
Step 1: if ~exist('show') show='ns'; end
Step 2: if ~exist('niter') niter=100; end
Step 3: if ~exist('K') K=15; end
Step 4: if ~exist('Io') Io=I; end
Step 5: if ~exist('dt') dt=0.2; end
Step 6:if (nargin<3) error(not enough arguments)
Step 7:if edgestop=='lin'
   k=1;
Step 8:elseif edgestop=='pm1'
   k=K;
   a=1;
Step 9:elseif edgestop=='pm2'
   k=K*(2^0.5);
   a=1/(2*exp(-0.5));
Step 10:elseif edgestop=='tky'
   k=K*5^0.5;
   a=25/32;
Step 11:if(show=='is') figure; end
```

Then it follows the four order iteration in the following ways as the above, then the row wise division is shown in the following manner:

```
Ix=(I(:,[2:col col],:)-I(:,[1 1:col-1],:))/2;%Ix=(E-W)/2 Iy=(I([2:row row],:,:)-I([1 1:row-1],:,:))/2;%Iy=(S-N)/2
Ixx=I(:,[2:col col],:)+I(:,[1 1:col-1],:)-2*I;%Ixx=E+W-2*O
Iyy=I([2:row row],:,:)+I([1 1:row-1],:,:)-2*I;%Iyy=S+N-2*O
ESWN=I([2:row row],[2:col col],:)+I([1 1:row-1],[1 1:col-1],:);%ES+WN
ENWS=I([1 1:row-1],[2:col col],:)+I([2:row row],[1 1:col-1],:);%EN+WS
```

Ixy=(ESWN-ENWS)/4;

5. Result Analysis

For result comparison we consider three different images leena and cameraman and show the effectiveness of our algorithm. For leena the noise parameter is 10 and for cameraman the noise parameter is 8 and 15. The results are shown in figure 1, figure 2 and figure 3.
The table 1 shows the comparison which shows the effectiveness of our approach.

Table 1: Result Analysis

Noise Parameter	Image with Noise	Mean Filter	PDE	BEMD and PDE	Proposed PDE
Leena (10)	14.7	18.5	19.5	21.1	23.5
Cameraman(15)	10.4	15.4	17.2	19.7	23.5
Cameraman(8)	19.6	19.9	21.1	23.1	24.1

6. Conclusion

In this paper we proposed an efficient approach based on fourth order PDE. A standard denoising technique is the convolutions of the image with different distribution technique. We provide here a comparison consider three different noise parameters and improves the SNR ratio, which reduces the noise and blur.

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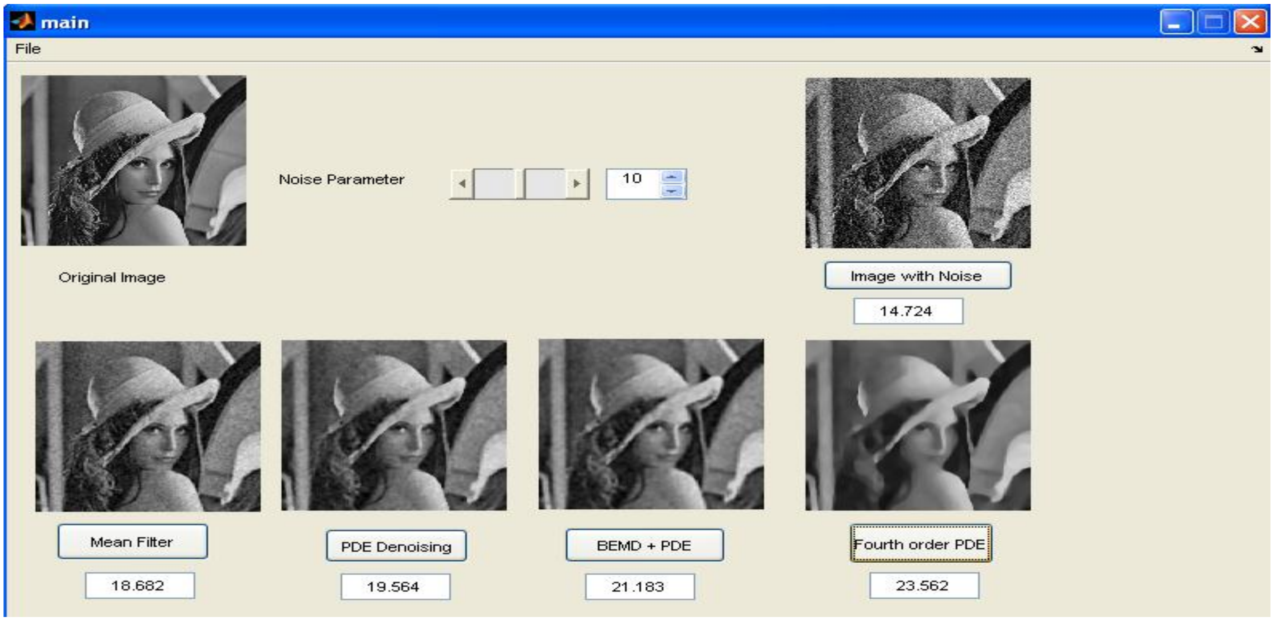

Figure 1: Leena Image Based SNR Comparison

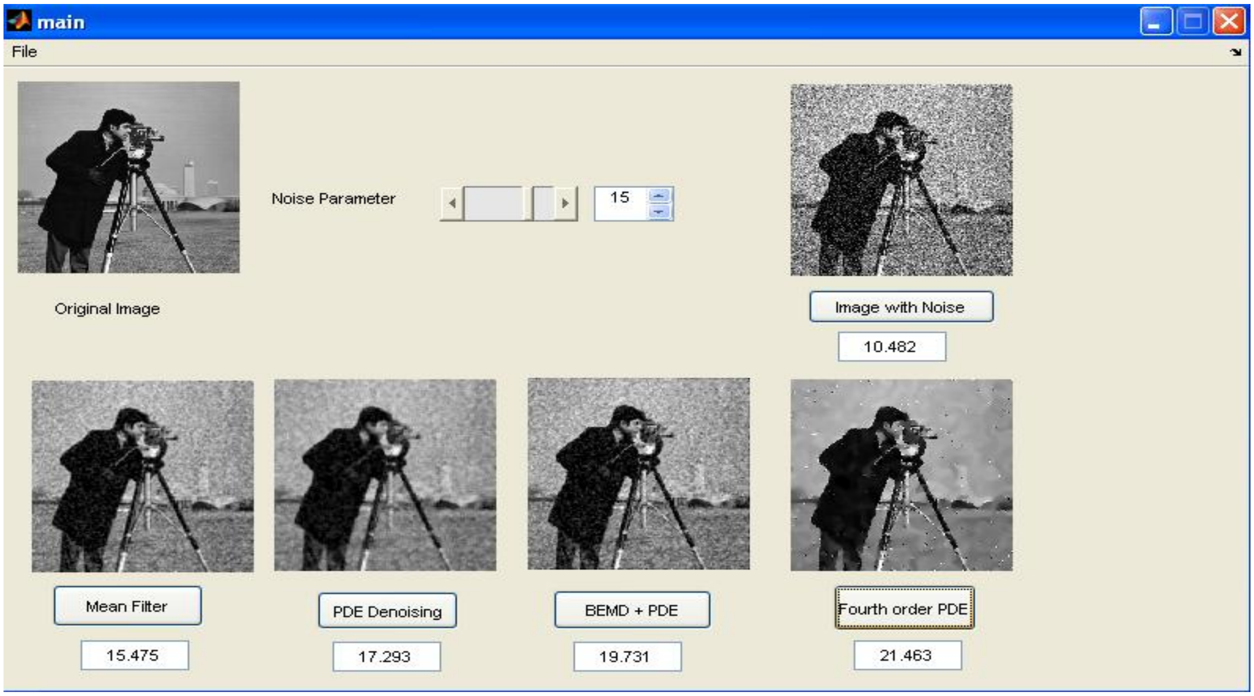

Figure 2: Cameraman Image Based SNR Comparison with Noise parameter (15)

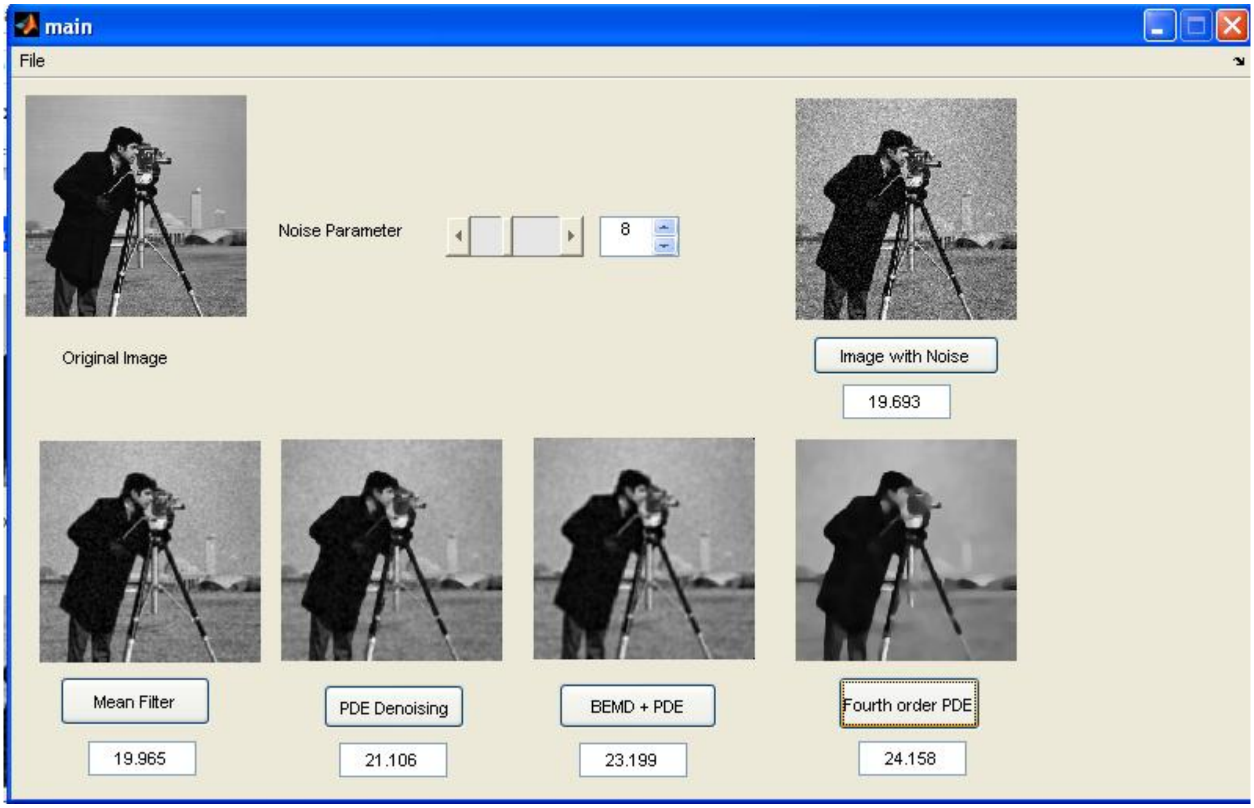

Figure 3: Cameraman Image Based SNR Comparison with Noise parameter (8)