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Intuitionistic fuzzy stability of cubic type mapping

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Abstract

Recently A.Alotaibi, M.Mursaleen , H. Dutta and S.A.Mohiuddine [1] proved the ulam stability of Cauchy functional equation f(x+y)=f(x)+f(y) in the intuitionistic fuzzy normed spaces. S.A.Mohiuddine and A. Alotaib i[2] proved the Fuzzy stability of a cubic functional equation via fixed point technique In this paper we prove the intuitionistic fuzzy stability of cubic functional equation f(2x+y)+f(2x-y)=2f(x+y)+2f(x-y)+12f(x) by using the fixed point alternative.

Keywords

Stability, Functional equation, intuitionistic fuzzy space, cubic mapping.

1. Introduction

A question in the theory of functional equations is the following "When is it true that a function which approximately satisfies a functional equation $\in$ must be close to an exact solution $\in$?" If the problem accepts a solution, than we say that the equation $\in$ is stable. S.M.Ulam [3] discussed the following question concerning the stability of homomorphism: "Let (G_1, *) be a group and (G_2, o,d) be a metric group with the metric d. Given $\in$ >0, does there exists a $\delta_\in$ >0 such that if a mapping h: $G_1 \rightarrow G_2$ satisfies the inequality d(h(x*y),h(x)o h(y))< $\delta_\in$ $\forall$ x,y$\in G_1$, then there is a mapping H: $G_1 \rightarrow G_2$ such that for each x,y $\in G_1$ H(x*y)=H(x) oH(y) and d(h(x),H(x))< $\in$?" In the next year, D.H.Hyers [4], gave answer to the above question for additive groups under the assumption that groups are Banach spaces. In 1978, T.M.Rassias [5] proved a generalization of Hyers' theorem for additive mapping as a special case in the form of following result.

Theorem 1.1[4]. Suppose that E and F are real normed spaces with F a complete normed space, f: E $\rightarrow$F is a mapping such that for each fixed x$\in$E the mapping t$\rightarrow$f (tx) is continuous on R, and let there exist ε >0 and p$\in$[0,1) s.t

$$\|f(x+y)-f(x)-f(y)\| \leq \varepsilon(\|x\|^p+\|y\theta\|^p)$$

x,y$\in$E.

Then there exist a unique linear mapping T:E$\rightarrow$F s.t

$$\|f(x)-T(x)\| \leq \varepsilon \frac{\|x\|^p}{(1-2^{p-1})}, \quad x\in E$$

The functional equation

f(2x + y) + f(2x – y) = 2f(x +y) +2 f(x –y) + 12f(x)

is said to be the cubic functional equation since cx^3 is its solution. The solution of the cubic functional equation is said to be cubic mapping. Another cubic functional equation is

f(x+y+2z)+f(x+y-2z)+f(2x)+f(2y)
=2f(x+y)+4f(x+z)+4f(x-z) +4f(y+z)+4f(y-z).

The stability problem for the cubic functional equation was proved by K.W.Jun and H.M.Kim [6] for mappings f: X $\rightarrow$Y where X is a real normed space and Y is Banach space and also provedthe stability for the functional equation f(2x + y) + f(2x – y) = 2f(x +y) +2 f(x –y) + 12f(x) in real vector spaces. The objective of paper is to prove the stability of this cubic functional equation in intuitionistic fuzzy normed space. S.M. Jung and T.S.Kim,[7], Chang, I.S., Jun, K.M. and Jung, Y.S [8], Y. Jung and I.Chang,[9] and A.K.Mirmostafaee and M.S.Moslehian[11] proved the stability of cubic functional equations in various spaces. Recently the stability of Jensen. Quadratic and mixed type additive cubic functional equations have been considered in [12], [13],[14] and[15].

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K.T.Atanassov [10] introduced the Intuitionistic fuzzy sets. The notion of intuitionistic fuzzy normed space was introduced by S.B.Hasseini, D.O'Regan and R.Saadati [16] as generalization of fuzzy normed space as:

Definition 2.1. Let μ and ν be the membership and the nonmembership degree of an intuitionistic fuzzy set from X $\times(0,+\infty)$ to [0,1] such that $\mu_x(t)+\nu_x(t)\le 1$ for all $x\in X$ and t>0. The triple $(X,P_{\mu,\nu},\tau)$ is said to be an intuitionistic fuzzy normed space (briefly IFN-space) if X is a vector space, τ is a continuous t-representable, and $P_{\mu,\nu}$ is a mapping $X\times(0,+\infty)\to L^*$ satisfying the following conditions: for all x,y$\in X$ and t, s>0,

(a) $P_{\mu,\nu}(x,0)=0_{L^*}$;

(b) $P_{\mu,\nu}(x,t)=1_{L^*}$ if and only if x=0;

(c) $P_{\mu,\nu}(ax,t)=P_{\mu,\nu}(x,t/a)$ for all $a\neq 0$;

(d) $P_{\mu,\nu}(x+y,t+s)\ge\tau(P_{\mu,\nu}(x,t),P_{\mu,\nu}(y,s))$

In this case, $P_{\mu,\nu}$ is called an intuitionistic fuzzy norm. Here, $P_{\mu,\nu}(x,t)=(\mu_x(t),\nu_x(t))$.

Example 2.2. Let (X, ||.||) be a normed space. Let τ(a,b)=(a_1b_1, min(a_2+b_2,1)) for all a=(a_1,a_2), b=(b_1,b_2) $\in L^*$ and μ,ν be membership and non-membership degree of an intuitionistic fuzzy set defined by

$$P_{\mu,\nu}(x,t)=(\mu_x(t),\nu_x(t))=\left(\frac{t}{t+\|x\|},\frac{\|x\|}{t+\|x\|}\right),$$

$$\forall t\in R^+$$

Then (X, $P_{\mu,\nu},\tau$) is an IFN-space.

Definition 2.3. A sequence $\{x_n\}$ in an IFN-space (X, $P_{\mu,\nu},\tau$) is called a Cauchy sequence if for any $\in$>0 and t>0, there exists $n_0\in$ N such that

$$P_{\mu,\nu}(x_n-x_m,t)>_{l^*}(N_s(\in),\in),\qquad \forall n,m\ge n_0$$

where N_s is the standard negator.

Definition 2.4. A sequence $\{x_n\}$ is said to be convergent to a point x$\in$X if

$P_{\mu,\nu}(x_n-x,t)\to 1_{L^*}$ *as* $n\to\infty$ for every t>0.

Definition 2.5. An IFN-space(X, $P_{\mu,\nu},\tau$) is said to be complete if every Cauchy sequence in X is convergent to a point x$\in$X.

2. Main Results

Theorem 3.1 Let X be a linear space, $(Z,P'_{\mu,\nu},M)$ an IFN-space, and $\phi: X\times X\to Z$ a function such that for some $0\le a\le 8$,

$$P'_{\mu,\nu}(\phi(2x,2y),t)\ge_{L^*}P'_{\mu,\nu}(a^2\phi(x,y),t)$$
(x,y$\in X$,t>0) (1)

$$\lim_{n\to\infty}P'_{\mu,\nu}(\phi(2^nx,2^ny),8^n/a^{2n}t)=1_{L^*}$$
(2)

for all x,y $\in X$ and t>0. Let $(Y,P_{\mu,\nu},M)$ be a complete IFN-space. If f:X$\to$Y is a mapping such that , for all x,y$\in X$,t>0

$$P_{\mu,\nu}(Df(x,y),t)\ge_{L^*}P'_{\mu,\nu}(\phi(x,y),t).$$
(3)

and f(0)=0, then there is a unique quadratic mapping C:X$\to$Y

$$P_{\mu,\nu}(f(x)-C(x),t)\ge_{L^*}P'_{\mu,\nu}(\phi(x,0),(16-2a)t).$$
(4)

Where, Df(x,y)=f (2x+y)+f(2x-y)-2f(x+y)-2f(x-y)-12f(x) for all x,y$\in X$. (*)

Prof: Put y=0 in (3), we get

$$P_{\mu,\nu}(2f(2x)-16f(x),t)\ge_{L^*}P'_{\mu,\nu}(\phi(x,0),t).$$

$$P_{\mu,\nu}(\frac{f(2x)}{8}-f(x),t)\ge_{L^*}P'_{\mu,\nu}(\frac{1}{16}\phi(x,0),t).$$
(5)

Replacing x= 2^nx in (5), we have

$$P_{\mu,\nu}(\frac{f(2^{n+1}x)}{8}-f(2^nx),t)\ge_{L^*}P'_{\mu,\nu}(\frac{1}{16}\phi(2^nx,0),t).$$

$$P_{\mu,\nu}(\frac{f(2^{n+1}x)}{8^{n+1}}-\frac{f(2^nx)}{8^n},t)\ge_{L^*}$$

$$P'_{\mu,\nu}(\frac{1}{16\times 8^n}\phi(2^nx,0),t).$$

$$P_{\mu,\nu}(\frac{f(2^{n+1}x)}{8^{n+1}}-\frac{f(2^nx)}{8^n},t)\ge_{L^*}P'_{\mu,\nu}(\frac{a^n}{16\times 8^n}\phi(x,0),t).$$
(6)

We write,

$$\frac{f(2^n x)}{8^n} - f(x) = \sum_{k=0}^{n-1} \frac{f(2^{n+1} x)}{8^{n+1}} - \frac{f(2^n x)}{8^n}$$ and from (6),

$$P_{\mu,\nu}\left(\frac{f(2^n x)}{8^n} - f(x), t\right) =$$

$$P_{\mu,\nu}\left(\sum_{k=0}^{n-1} \frac{f(2^{k+1} x)}{8^{k+1}} - \frac{f(2^k x)}{8^k}, t\right)$$

$$\geq_{L^*} P'_{\mu,\nu}\left(\sum_{k=0}^{n-1} \frac{a^k}{16 \times 8^k} \phi(x,0), t\right). \quad (7)$$

Replacing x with $2^m x$ in (6), we observe

$$P_{\mu,\nu}\left(\frac{f(2^{n+m} x)}{8^{n+m}} - \frac{f(2^m x)}{8^m}, t\right)$$

$$\geq_{L^*} P'_{\mu,\nu}\left(\sum_{k=m}^{n+m-1} \frac{a^k}{16 \times 8^k} \phi(x,0), t\right).$$

(8)

Then $\{f(2^n x)/8^n\}$ is a Cauchy sequence in $(Y, P_{\mu,\nu}, M)$. Since $(Y, P_{\mu,\nu}, M)$ is a complete IFN-space, this sequence converges to some point C(x) $\in Y$. Put m=0 and fix $x \in X$ in (8), we get

$$P_{\mu,\nu}\left(\frac{f(2^n x)}{8^n} - \frac{f(x)}{}, t\right)$$

$$\geq_{L^*} P'_{\mu,\nu}\left(\sum_{k=0}^{n-1} \frac{a^k}{16 \times 8^k} \phi(x,0), t\right).$$

and

$$P_{\mu,\nu}(C(x) - f(x), t)$$

$$\geq \tau\begin{pmatrix} P_{\mu,\nu}\left(C(x) - \frac{f(2^n x)}{8^n}, t\right), \\ P_{\mu,\nu}\left(\frac{f(2^n x)}{8^n} - f(x), t\right), \end{pmatrix}.$$

$$\geq \tau\begin{pmatrix} P_{\mu,\nu}\left(C(x) - \frac{f(2^n x)}{8^n}, t\right), \\ P'_{\mu,\nu}\left(\sum_{k=0}^{n-1} \frac{a^k}{16 \times 8^k} \phi(x,0), t\right) \end{pmatrix}.$$

Taking $n \to \infty$, we get

$$P_{\mu,\nu}(C(x) - f(x), t) \geq_{L^*} . P'_{\mu,\nu}\left(\frac{\phi(x,0)}{16 - 2a}, t\right).$$

Thus

$$P_{\mu,\nu}(f(x) - C(x), t) \geq_{L^*} P'_{\mu,\nu}(\phi(x,0), (16 - 2a)t).$$

Now show that C(x)satisfies (*)

Replacing x by $2^n x$ and y by $2^n y$ in (3),we have

$$P_{\mu,\nu}\begin{pmatrix} f(2^n(2x+y)) + f(2^n(2x-y)) \\ -2f(2^n(x+y)) - 2f(2^n(x-y)) \\ -12 f(2^n x), t \end{pmatrix}$$

$$\geq_{L^*} P'_{\mu,\nu}(\phi(2^n x, 2^n y), t).$$

$$P_{\mu,\nu}\begin{pmatrix} f(2^n(2x+y))/8^n + f(2^n(2x-y))/8^n \\ -2f(2^n(x+y))/8^n - 2f(2^n(x-y))/8^n \\ -12 f(2^n x)/8^n, t \end{pmatrix}$$

$$\geq_{L^*} P'_{\mu,\nu}(a^{2n}\phi(x,y), t8^n).$$

Since $\lim_{n\to\infty} P'_{\mu,\nu}(\phi(2^n x, 2^n y), 8^n / a^{2n} t) = 1_{L^*}$,

we conclude that C fulfils (*).

To Prove the uniqueness of mapping C, Let there exists D: X $\rightarrow$ Y which satisfies (4). Clearly $C(2^n x) = 8^n C(x)$ and $D(2^n x) = 8^n D(x)$ for all $n \in$ N. Then

$$P_{\mu,\nu}(C(x) - D(x), t)$$

$$= \lim_{n\to\infty} P_{\mu,\nu}(C(2^n x/8^n) - D(2^n x/8^n), t)$$

$$P_{\mu,\nu}(C(2^n x/8^n) - D(2^n x/8^n), t)$$

$$\geq_{L^*} P'_{\mu,\nu}(\phi(2^n x, 0), 8^n(8 - \alpha)t)$$

$$\geq_{L^*} P'_{\mu,\nu}\left(\phi(x,0), \frac{8^n(8-\alpha)t}{\alpha^n}\right)$$

Since $\lim_{n\to\infty}\left(\frac{8^n(8-\alpha)}{\alpha^n} t\right) = \infty$, we get

$\lim_{n\to\infty} P'_{\mu,\nu}\left(\phi(x,0), \frac{8^n(8-\alpha)t}{\alpha^n}\right) = 1$. Therefore, it follows $P_{\mu,\nu}(C(x) - D(x), t) = 1$ for all t>0 so, C(x)=D(x).

Corollary3.2. let X be a linear space $(Z, P'_{\mu,\nu}, M)$ an IFN-space be complete IFN-Space, p,q be nonnegative real numbers and let $z_0 \in$ Z. if f:X $\rightarrow$ Y is a mapping such that

$$P_{\mu,\nu}\begin{pmatrix} f(2x+y)+f(2x-y) \\ -2f(x+y)-2f(x-y) \\ -12f(x),t \end{pmatrix}$$

$$\geq_{L^*} P'_{\mu,\nu}((\|x\|^p+\|y\|^q)z_0,t$$

x,y $\in$ X,t>0, f(0)=0 and p,q<1, then there exist a unique cubic mapping C:X $\rightarrow$ Y such that

$$P_{\mu,\nu}(f(x)-C(x),t)\geq_{L^*} P'_{\mu,\nu}((\|x\|^p)z_0,(8-8^p)t)$$

for all x $\in$ X and t>0.

Proof. Let $\phi: X\times X \rightarrow Z$ be defined by ϕ (x,y)=($\|x\|^p+\|y\|^q$)z_0. Then the corollary followed from the theorem 3.1 by α =2^p.

3. Conclusion

In this paper, we have developed the stability of cubic type mapping in Intuitionistic fuzzy normed linear space using the fixed point theory.

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