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APM (Simple MPC) vs. PID – Detailed Comparison

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Abstract

In this paper APM (Asymptotic Predictive Method) algorithm for real time control of LTI SISO (Linear Time Invariant Single Input Single Output) systems has been compared with PID algorithm on several statistically most common real SISO models in practice. APM is specific MPC (Model Predictive Control) algorithm which defines optimal control signal using trivial sequence of counter control signals, instead of demanding QP (Quadratic Programming) method. Using Matlab simulation, APM algorithm has been compared with PID algorithm. Comparison criteria are: mean absolute error, steady-state error, settling time, overshoot, disturbance rejection and robustness. In most of simulation examples, APM algorithm has provided better control performance than PID algorithm.

Keywords

PID, MPC, APM, control, linear, algorithm, disturbance.

1. Introduction

Over 70 years, PID algorithm is the most widely applied control algorithm. Due to its simplicity and efficiency, PID algorithm is applied to more than 80% of SISO plants. All other control strategies, including MPC control, participate with 20% only in regard to application in practice. Standard MPC control is superior to PID control in case of control of plants with output constraints as well as MIMO (Multiple Input Multiple Output) systems. Due to its high software and hardware demands and high implementation costs, did not endanger PID algorithm dominance in control of most common real LTI SISO plants or processes. Standard MPC control, due to its deficiency, is used only when inexpensive PID control can not provide requested control performance. Therefore creation of a practical and simple MPC algorithm is ongoing. Different MPC algorithms were created to decrease main deficiencies of standard MPC control and increase competitiveness in regard to dominant PID control.

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In paper [1] using certain approximation, modified MPC algorithm is developed that insures faster definition of optimal control signal with similar hardware and software demands as standard MPC algorithm. Also developed is explicit MPC [2] which stores optimal control signals to database in off line mode. In paper [3] PID algorithm is compared with a specific MPC algorithm based on orthonormal functions. Most of suggested modified MPC algorithms use demanding QP method to define optimal control signal, which as a result, has high software and hardware demands and expensive implementation [4]. APM algorithm [5] defines optimal control signal by specific counter control signal using characteristics of LTI systems: homogeneity and superposition. Therefore, APM algorithm could be routinely applied to LTI SISO plants using the same programmable controllers as PID algorithm.

2. Problem formulation

The problem of control in LTI SISO system is usually formulated in discrete domain, in state space:

$$x(k+1) = Ax(k) + Bu(k) + Ew(k) \qquad (1)$$

$$y(k) = Cx(k) \qquad (2)$$

Where:
x(k) – State variables vector
u(k) – Input variables vector
y(k) – Output variables vector
w(k) – Bounded disturbances vector
A,B,C,D,E – Appropriate system matrix

Noise measurement influence is usually neglected [6]. Constraints are always present because of actuator constraints [7] and usually given like:

$$U_{\min} \le u(k) \le U_{\max} \quad and \quad -\Delta U_{\max} \le \Delta u(k) \le \Delta U_{\max} \qquad (3)$$

$$Y_{\min} \le y(k) \le Y_{\max} \qquad (4)$$

Control objective is always the same: controlling the system according to defined criteria while respecting input and output constraints.

3. APM algorithm for LTI SISO systems

APM algorithm defines control signal on each discrete interval *kT*, using goal function *J*(*k*), by grading each potential control signal from the discrete set of possible inputs:

$$J(k) = \sum_{j=Hw}^{Hp} \left[r - \hat{y}\ (k + j|k)\right]^2 + \hat{P}^r \tag{5}$$

where:

$\hat{y}(k+j|k)$ - Output prediction at the moment kT

r – Given setpoint value (constant)
H_w– First prediction horizon (delay horizon)
H_p – Prediction horizon
P^r – Penalty of setpoint overshoot
Setpoint overshoot penalties are calculated as: (6)

$$\hat{P}^r = C^z \sum_{j=H_w}^{H_p} \left[r - \hat{y}(k+j|k)\right]^2 \left[1 - \lambda Sign(\hat{y}(k+j|k) - r)\right]$$

$$\lambda = Sign(y_0 - r) \tag{7}$$

where y_0 is measured output, r setpoint and λ is a sign of control error in a moment kT. By counter control correction

$$\Delta u^+(k) = \alpha\lambda\Delta U_{\max}\ , \quad 0 \prec \alpha \prec 1 \tag{8}$$

for each potential control signal $u_j(k)$ prediction control sequence $U_j(k)$ of length H_p is formed:

$$\hat{U}_q(k) = [\ \hat{u}_q(k), \hat{u}_q{}^+(1), \hat{u}_q{}^+(2), ..., \hat{u}_q{}^+(H_p - 1)\ ] \tag{9}$$

$$U_{\min} \le \hat{u}_q^+(n) \le U_{\max}\ , \quad 1 \le n \le H_{p-1}, \quad n \in N \tag{10}$$

where:

$$\begin{aligned} \hat{u}_q^+(1) &= \hat{u}_q(k) + \Delta u^+(k) \\ \hat{u}_q^+(2) &= \hat{u}_q^+(1) + \Delta u^+(k) \\ &\ldots \\ \hat{u}_q^+(H_p - 1) &= \hat{u}_q^+(H_q - 2) + \Delta u^+(k) \end{aligned} \tag{11}$$

Coefficient α defines decrement or increment rate of counter control sequence. Higher coefficient α provides more aggressive control. For plants highly sensitive to input changes, lesser value of coefficient α is adopted. In case of inverse systems, coefficient α has negative sign. Every potential control signal $u_q(k)$ is extended with its unique predictive counter-control sequence and evaluated by $J(k)$. For example, in case of positioning a car, sequence of counter-control signals is used to stop the car on the setpoint position. Signal $u_q(k)$ whose predictive output trajectory Y_q gets lowest grade, using goal function $J(k)$, is adopted as optimal control signal $u^*(k)$. The same procedure is repeated on each discrete interval.

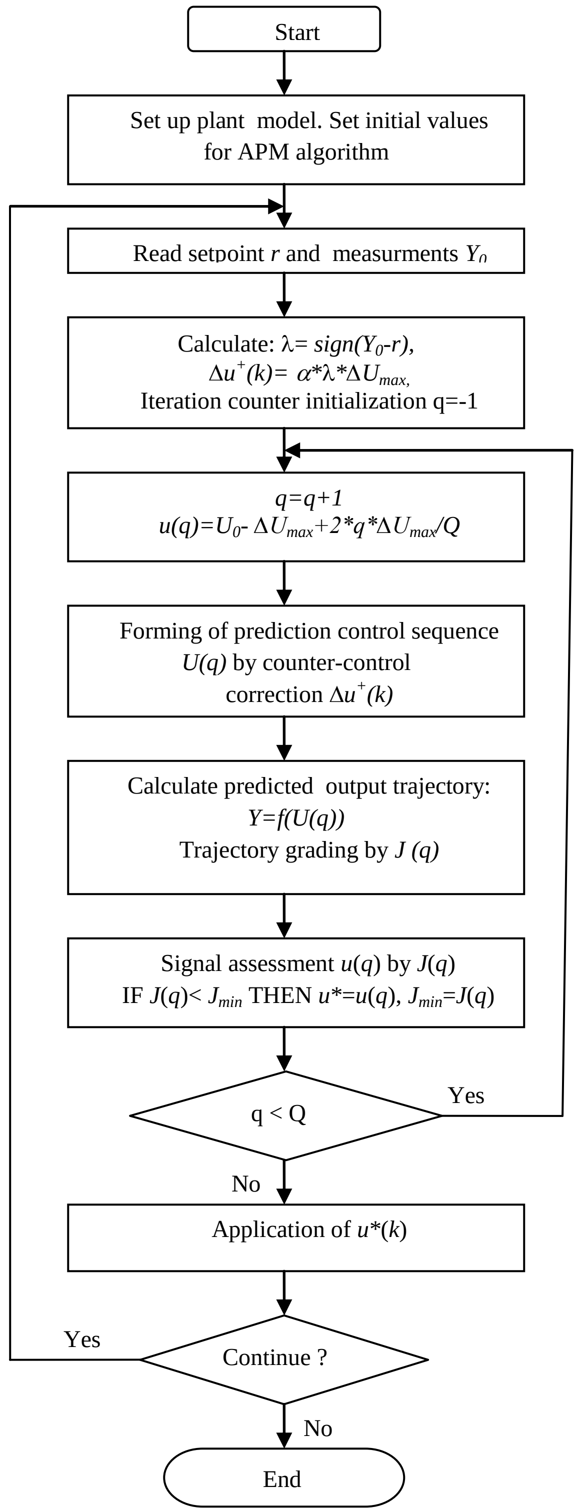

Figure 1: APM algorithm flow diagram

4. PID algorithm

Widely applied well known PID algorithm, generates control signal $u(k)$ based on control error $e(k)$ [8]. PID controller parameters are adjusted with an appropriate method in order to achieve desired control performance [9]. Most commonly, parallel PID is used whose output is calculated as:

$$U(s) = K_p\left(1+\frac{T_d s}{T_r s+1}+\frac{1}{T_i s}\right)E(s) \quad (12)$$

where:
U - PID Controller output
K_p - Proportional gain
T_d - Derivative time constant
T_r - Real differentiator time constant
T_i - Integral time constant

5. APM and PID simulation results

Comparison of APM and PID control on several typical, practically most common models was completed by Matlab simulation. Parameter adjustment of PID algorithm was performed by step response optimization (SROPT), IMC (Internal Model Control), Robust PID and Switched PID methods, while APM parameter adjustment was completed by simulation method, based on step response. Comparative simulation results of control are given graphically and in a table.

Table 1: Control results comparison

Algorithm	$\|e\|$	$t_s\,(s)$	Overshoot (%)
APM	0.07	22	0
PID	0.09	40	12

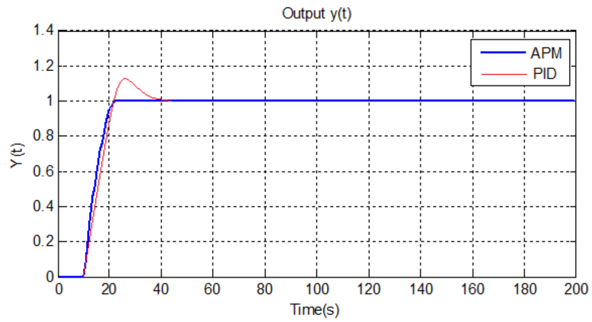

Figure 2: APM and PID control of FOPDT plant

5.1 Control of first order plant with time delay

Statistically, the most real control plants (or processes) are modeled by first order system with or without time delay (FOPDT). Given a plant to be controlled whose transfer function and input constraints are defined as [3]:

$$G_1 = \frac{e^{-10s}}{\tau s+1} = \frac{e^{-10s}}{10s+1} \quad and \quad -1.5 \le u \le 1.5 \quad (13)$$

Sampling period is: T=0.5 s. Figure 2 shows simulation of FOPDT plant control using PID and APM algorithm. PID algorithm parameters are set by step response optimization (SROPT) method. Parameters of APM algorithm are set by simulation method, based on step response. Figure 2 shows output trajectories at APM and PID control of plant FOPDT [3]. Table 1 shows basic metric parameters of transitional processes in APM and PID control. APM control is more qualitative because it provides shorter settling time, lesser overshoot and lesser control error. Figure 3 shows control simulation of the compared algorithms with model error. Time constant of FOPDT plant is increased for 50% (τ=15s). Due to higher time constant τ, settling time is increased. Simulation results show that APM algorithm is more robust because control performance is not significantly disrupted. Table 2 shows basic metric parameters of APM and PID simulation control of FOPDT plant with model error. Based on comparison of APM algorithm with (SROPT) PID algorithm, it is conclusive that APM algorithm ensures more qualitative control of FOPDT plants.

Table 2: Control results comparison

Algorithm	$\|e\|$	$t_s\,(s)$	Overshoot (%)
APM	0.09	28	0
PID	0.10	72	11

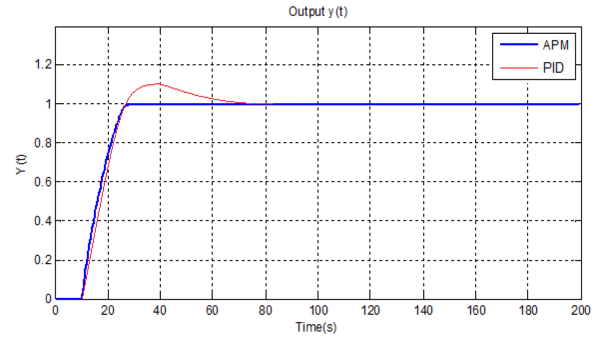

Figure 3: APM and PID control of FOPDT with 50 percent error in time constant τ in process model

5.2 Control of second order plant SOPDT

Given a second order plant with time delay whose transfer function and input constraints [10]:

$$G_1 = \frac{e^{-30s}}{(1+10s)(1+10s)} \quad , \quad -1.5 \le u \le 1.5 \qquad (14)$$

Sampling time is: T=1s. Delay time is 30s. SOPDT model is very often used in practice. APM algorithm is compared with PID algorithm whose parameters are adjusted by IMC method. APM algorithm parameters are adjusted based on step response of the plant (14). Generally, IMC method of setting PID algorithm parameters ensures high control performance in case of lower order plant with time delay. IMC method of setting PID algorithm parameters ensures better performance of SOPDT plant control than ZN (Ziegler Nichols) method. The time delay term is approximated for simplification of mathematical model of a controlled plant. Figure 4

Table 3: APM and PID control results comparison

Algorithm	$\|e\|$	$t_s\,(s)$	Overshoot (%)
APM	0,11	60	0
PID	0,13	160	8

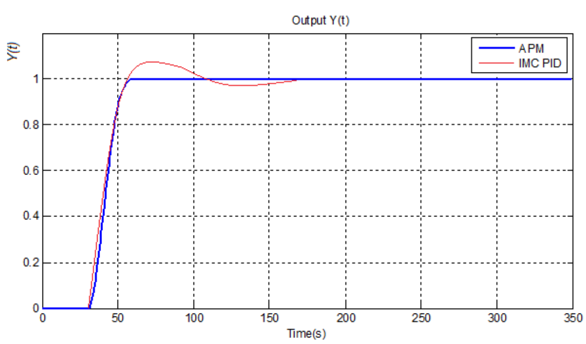

Figure 4: APM and IMC PID control of SOPDT

shows comparative results of APM and IMC PID control of SOPDT plant (14). Based on measurable metric parameters of step response given in table 3, derives that APM algorithm ensures better control performance. APM algorithm ensures shorter settling time and lesser control error. Figure 4, graphically shows simulation results of APM and IMC PID plants control (14). APM algorithm generates more qualitative control because it ensures lesser control error without overshoot. PID control generates control with higher overshoot and longer settling time.

5.3 Control of nonminimum phased plant

Control of nonminimum phase plants is demanding due to the inversion of the initial part of the transitional process. Response inversion is happening due to influence of zeros in the right s-half plane. Depending on the position and number of zeros in the right s-half plane, inversion could be more or less expressed. Plants with zeros in the right s-half plane which are located near the origin could be very difficult to control, due to sensitivity to input changes and disturbances. In the example (15) of non-minimum phase second order plant with one finite zero, APM and PID control was compared. PID algorithm parameters are adjusted by IMC method [11], while APM algorithm parameters are adjusted by a simulation method based on step response.

Table 4: APM and IMC PID Control results

Algorithm	$\|e\|$	t_s	Overshoot (%)
APM	0,14	14	12
PID	0,15	13.5	6

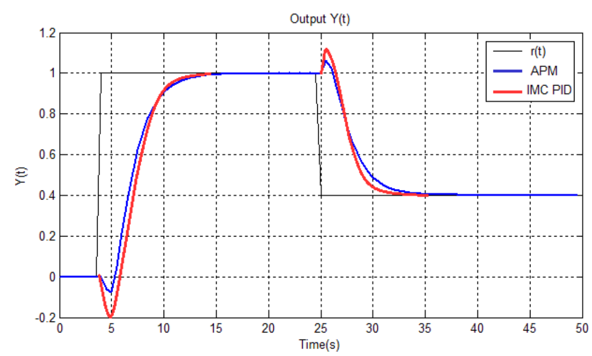

Figure 5: APM and IMC PID Control

IMC method implies use of referent plant model for prediction of model behaviour. Given transfer function of the plant and input constraints [11]:

$$G_1 = \frac{2(1-s)}{(2s+1)(s+1)} \quad \textit{and} \quad -1 \le u \le 1 \qquad (15)$$

Table 4 shows basic metric parameters of concurrent trajectories in a controlled output. APM and PID control performances are similar. PID control ensures shorter settling time, while APM control ensures lesser overshoot and lesser control error. Figure 5 graphically shows simulation results of APM and PID control. Output trajectories of APM and PID controls are similar. PID control is more energetic because it ensures shorter settling time but higher overshoot as well.

5.4 Control of third order plant (PID 3)

Given the control task to control pitch angle of an Unmanned Aerial Vehicle (UAV), providing overshoot lesser than 2% and settling time lesser than 2 *s*. The plant transfer functions given [12]:

$$G = \frac{21.9976}{0.81698s^3 + 0.89665s^2 + 1.0275s + 1} \qquad (16)$$

Table 5: APM and PID control results comparison

Algorithm	$\|e\|$	$t_s\,(s)$	Overshoot (%)
APM	0,08	2,2	0.5
PID	0,09	1,9	0

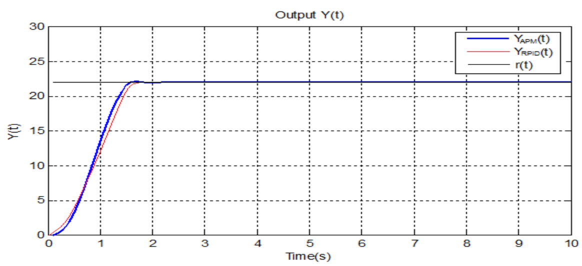

Figure 6: APM and robust PID Control

Input constraints given:

$$-10 \le u \le 10 \qquad (17)$$

For a steady state flight, the transfer function (16) represents an acceptable approximation of the relationship between pitch angle and elevator angle. APM and PID control of a plant is simulated. Sampling time is T=0.1*s*. PID parameters have been adjusted using Robust PID controller design. APM algorithm parameters have been adjusted by a simulation method, based on step response of a plant. Table 5 shows basic metric parameters of simulated results in APM and PID controls. APM ensures lesser control error with a low overshoot of 0.5 %. On the other side PID control ensures shorter settling time. Figure 6 shows simulation results of APM and Robust PID control. Trajectories of APM and PID controlled output are almost identical so the quality of control is similar. Both algorithms ensure qualitative control without static error, respecting given requirements in regard to overshoot and settling time.

5.5 Disturbance rejection

The lesser sensitivity to disturbance, the control is more qualitative [13]. In the example of SOPDT plant (18) sensitivity to disturbance has been analyzed of APM and PID algorithms. Plant transfer function and input constraints given [14]:

$$G = \frac{e^{-0.5s}}{(s+1)^2} \qquad (18)$$

$$-10 \le u \le 10 \qquad (19)$$

APM and PID control - positioning to a set point value has been simulated. PID parameters were adjusted using Switched PID method which ensures small overshoot and short settling time with acceptable disturbance compensation. APM algorithm parameters were adjusted by the plant's step response. In a moment t=10*s*, a singular negative disturbance was emitted to test how concurrent algorithms compensate the influence of such disturbance. For sampling time T=0.1*s*. APM algorithm generates a steady state error of 2% while in a shorter sampling period T=0.005*s* (Figure 7), APM control eliminates static error and efficiently compensates the influence of negative singular disturbance (Table 6). Based on shown simulation results of APM and PID control, it is conclusive that APM algorithm, for short sampling time T, ensures concurrent results in regard to disturbance compensation.

Table 6: APM and PID control results comparison

Algorithm	$\|e\|$	$t_s\,(s)$	Overshoot (%)
APM	0,08	3	-3
PID	0,12	4	-25

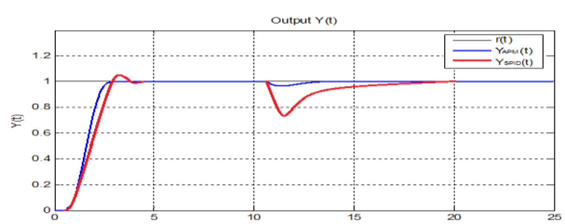

Figure 7: APM and switched PID disturbance rejection

6. Conclusion

Based on shown simulation results, it is conclusive that APM algorithm ensures better control performance in most of examples statistically most common real plants. On the other side, PID algorithm requires lesser measurement and it can reach higher control speed. Due to its source code compactness (5 KB), APM algorithm could be implemented on the same hardware as the PID algorithm (PLC, SoftPLC etc.) without any additional software. Therefore APM algorithm could be used as efficient and cheap MPC replacement for PID algorithm.

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