International Journal of Advanced Computer Research (ISSN (print): 2249-7277 ISSN (online): 2277-7970)
Volume-4 Number-2 Issue-15 June-2014

Performance Analysis of Non- Coherent FSK System with Square Law Combining our Nakagami- m Fading Channel

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Abstract

The probability of error for NFSK with square law combining for diversity correlated Nakagami channels is analyzed. The closed for of expression using characteristic function is derived. The average SNR per branch is considered to be distinct. The effect of different noise powers for various diversity orders and fading channel parameters on probability of error and unequal SNR distribution is being studied. All calculation has been made for total SNR and the average SNR for each diversity branch is achieved from total SNR. [9]

Keywords

NFSK, Square Combiner, SNR and Fading channel.

1. Introduction

As we know, the multi channel signalling is fixed channels differing in attenuation and phase shift. The propagation of signal on the channel depends on the characteristics of channel. There are multiple propagation paths for a channel. Each path is associated with a propagation delay and an attenuation factor which are time variant due to change in the structure of medium. The amplitude variation on the received signal due to time variant characteristics of the channel is called signal fading. The time variant impulse response of it is modelled by Rayleigh distribution, Rican fading channel and Nakagami fading channel respectively. Due to fading, the signal on the channel is severely degraded. To check up such degradation diversity techniques have been employed in which the several replicas of the same information bearing signal are supplied to the receiver on independent fading channels with the probability that at least one of the signal is not deeply faded. Space diversity reception is a well-known technique for combating the effects of fading in wireless communication system [1]. For space diversity schemes, three combining techniques namely, Maximum Ratio Combining (MRC), Equal Gain Combining (EGC) and selection Combining (SC) [2] are employed. Post detection equal gain combining is an effective means of combating multi path fading in non coherent MFSK systems. System design depends on the criterion of symbol error probability.

The error performance of non coherence MFSK with square law combing is available for correlated Nakagami [3] and correlated Rician [4] independent fading channels. The closed form solution for symbol error probability for NMFSK using characteristics function of combiner output SNR for equal gain combiner, is presented by Win and Malik [5] and Digham ao Alouini [6] for Nakagami and Rician fading channels. In this paper we considered NMFSK with square law combiner and L-diversity for Nakagami channels. We drive the general solution for NMFSK for Nakagami fading channels for symbol error probability for complex additive white Gaussian noise (AWGN) with zero mean different variances. We have considered the total SNR and different average SNR for different branches at the output of combiner. The organization of paper is constructed as: section 2^{nd} contains the formulation of problem and, section 3^{rd} contains the derivation of closed form expression for SEP. Result and discussions are shown in section 4^{th}. Finally, the conclusion of the discussion is derived in section 5^{th}.

2. The Formulation of System Model:

The signal received on k^{th} diversity branch for k= 1, 2...L corrupted by AWGN can be expressed as

$$r_k(t) = \mathrm{Re}\left\{\left[\alpha_k \exp(-j\varphi_k)s(t) + n_k(t)\right]\exp(j2\pi f_c t)\right\}, 0 \le t \le T_s$$

(T_s is symbol period).

Where, s (t) is complex signal containing information. α_k and φ_k are complex random fading amplitude and phase respectively. $n_k(t)$ is complex white Gaussian noise with zero mean and different

Manuscript received April, 21,2014.
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variances $\{n_k(t)\}$ are statistically independent. Following [1] the decision variables for M-ary orthogonal signals with square law combiner on the L channels can be expressed as

$$u_1 = \sum_{k=1}^{L} \left|2\varepsilon\alpha_k + n_{k1}\right|^2$$

$$u_m = \sum_{k=1}^{L} \left|n_{km}\right|^2, \text{ m=2, 3,...M}$$

The variance $\sigma_k{}^2 = \frac{1}{2} E\left(\left|n_{km}\right|^2\right) = 2\varepsilon N_k$

k= 1, 2...L,

Thus, we consider distinct values of noise power ε is the energy of signal $\{u_m\}$, m=2, 3,....M are statistically independent and identically chi-square distributed random variables, each with 2L degree of freedom.[7-8]

Here $u_2 < u_1, u_3 < u_1, \ldots\ldots u_m < u_1$

The total SNR at the combiner output is given by

$$\gamma_T = \frac{\sum_{k=1}^{L} \left|2\varepsilon\alpha_k\right|^2}{\sum_{k=1}^{L} \sigma_k{}^2}, \sigma_1 \neq \sigma_2 \neq \sigma_3 \ldots\ldots\ldots\ldots \neq \sigma_L$$

$$= \frac{\sum_{k=1}^{L} \left(2\varepsilon\alpha_k\right)^\dagger \left(2\varepsilon\alpha_k\right)}{\sigma_T{}^2}, \sigma_T{}^2 = \sigma_1{}^2 + \sigma_2{}^2 + \ldots\ldots\ldots \sigma_L{}^2$$

$$= \frac{\sum_{k=1}^{L} g_k{}^\dagger g_k}{\sigma_T{}^2}$$

$$= \frac{g^\dagger g}{\sigma_T{}^2} \qquad (1)$$

Where $g_k = 2\varepsilon\alpha_k$, the complex gain of k[th] channel and $g = \left[g_1, g_2, \ldots\ldots g_L\right]^T$ is gain vector and $g^\dagger$ is its Hermitian transpose.

The decision variables u_1 and u_2 are distributed according to a Chi-square probability distribution with 2L degrees of freedom.[10]

So,

$$P\left(u_2\right) = \frac{1}{2\sigma_T{}^2 (L-1)!} u_2{}^{L-1} \exp\left[\frac{-u_2}{2\sigma_T{}^2}\right] \quad (2)$$

The probability density function is P $(u_2<u_1)$ is given by

$$P\left(u_2 < u_1\right) = \int_0^{u_1} P\left(u_2\right) du_2 \qquad (3)$$

Substituting eqn. (2) into eqn. (3) and integration, we have

$$P\left(u_2 < u_1\right) = 1 - \exp\left(\frac{-u_1}{2\sigma_T^2}\right) \sum_{k=0}^{L-1} \frac{u_1^k}{k!} \left(\frac{1}{2\sigma_T^2}\right)^k \quad (4)$$

The probability density P $(u_2<u_1)$ is raised to (M-1) power to get the conditional probability of u_1

$$\left[P(u_1)\right]^{M-1} = \left[1 - \exp\left[\frac{-u_1}{2\sigma_T^2}\right] \sum_{k=0}^{L-1} \frac{u_1^k}{k!} \left[\frac{1}{2\sigma_T^2}\right]^k\right]^{M-1} \quad (5)$$

$$= 1 + \sum_{m=1}^{M-1} (-1)^m \frac{(M-1)!}{(M-m-1)!m!} \times \exp\left(\frac{-mu_1}{2\sigma_T^2}\right) \left[\sum_{k=0}^{L-1} \frac{u_1^k}{k!} \left(\frac{1}{2\sigma_T^2}\right)^k\right]^m \quad (6)$$

The average of P (u_1) can be obtained by invoking multinomial expansion [15] of power term, we get

$$\left[P(u_1)\right]^{M-1} = 1 + \sum_{m=1}^{M-1} (-1)^m \frac{(M-1)!}{(M-m-1)!} \sum_{m_0+m_1+\ldots\ldots+m_{L-1}=m} \alpha u_1^p \left(\frac{1}{2\sigma_T^2}\right)^k \exp\left(\frac{-mu_1}{2\sigma_T^2}\right) \quad (7)$$

Where

$$\alpha = \left[\prod_{k=1}^{L-1} m_k!(k!)^{m_k}\right]^{-1}$$

$$p = \sum_{k=0}^{L-1} km_k, \mathrm{m}_k \in \{0,1,2,\ldots\ldots \mathrm{m}\} \qquad (8)$$

3. Symbol Error Probability

By taking the average of eqn. (7), the conditional symbol error probability can be derived

$$P_c = <\left[P(u_1)\right]^{M-1}> = 1 + \sum_{m=1}^{M-1} (-1)^m \frac{(M-1)!}{(M-m-1)!m!} \sum_{m_0+m_1+\ldots\ldots\ldots+m_{L-1}=m} \alpha E\left[\left(\frac{u_1}{2\sigma_T^2}\right)^p \exp\left(\frac{-mu_1}{2\sigma_T^2}\right)\right] \quad (9)$$

Let

$$\frac{u_1}{2\sigma_T^2} = z$$

So,

$$z^p \exp(-mz) = z^p \exp(+sz), \mathrm{s} = -\mathrm{m}$$

$$E\left[z^p \exp(sz)\right] = E\left[\frac{d^p}{ds^p} \exp(sz)\right]$$

$$= \frac{d^p \Psi_z(s)}{ds^p}$$

$$\Psi_z(s) = E\left[\exp(sz)\right] \qquad (10)$$

S is a variable in transform domain
So, probability of error

$$P_e = 1 - P_c$$

$$= \sum_{m=1}^{M-1} (-1)^m \frac{(M-1)!}{(M-m-1)!m!} \sum_{m_0+m_1+\dots\dots+m_{L-1}=m} \alpha \frac{d^p \psi_z(s)}{ds^p} \quad (11)$$

Since

$$z \sim \frac{1}{2} \chi^2 (2L, 2\gamma_T) \quad (12)$$

Where $2\gamma_T = \frac{2g^\dagger g}{\sigma_T^2}$ is non-coherent parameter, and χ^2 indicates Chi-square distribution. Therefore, the conditional characteristic function of z is given as-

$$\psi_z(s|g) = (1 - 2s\sigma_T^2)^{-L} \exp\left(\frac{2s\sigma_T^2 \gamma_T}{1 - 2s\sigma_T^2}\right) \quad (13)$$

Thus, conditional characteristics function depends on the channel gain only through the total SNR γ_T and total variance σ_T^2.

So, $\psi_z(s|g) \sim \psi_z(s|\gamma_T)$

Replacing $\psi_z(s)$ by $\psi_z(s|\gamma_T)$ from eqn. (11) and substituting eqn. (13) into it, we have probability of error as

$$P_e(\gamma_T) = \sum_{m=1}^{M-1} (-1)^{m+1} \frac{(M-1)!}{(M-m-1)!m!} \sum_{m_0+m_1+\dots\dots+m_{L-1}=m} \alpha \frac{d^p}{ds^p} \left[(1-2s\sigma_T^2)^{-L} \exp\left(\frac{2s\sigma_T^2 \gamma_T}{1-2s\sigma_T^2} \right) \right]_{s=-m} \quad (14)$$

Where $p = m_1 + 2m_2 + \dots\dots + (L-1)m_{L-1}$.

Therefore, the average SEP is given by

$$P_e = E\left[P_e(\gamma_T) \right]$$

$$= \sum_{m=1}^{M-1} (-1)^{m+1} \frac{(M-1)!}{(M-m-1)!m!} \sum_{m_0+m_1+\dots\dots+m_{L-1}=m} \alpha \frac{d^p}{ds^p} \left[(1-2s\sigma_T^2)^{-L} \psi_{\psi_T} \left(\frac{2s\sigma_T^2}{1-2s\sigma_T^2} \right) \right]_{s=-m} \quad (15)$$

Where $\psi_{\psi_T}\left(\frac{2s\sigma_T^2 \gamma_T}{1-2s\sigma_T^2} \right)$ is the characteristic function of ψ_T.

This is the general formula for the SEP for arbitrary fading channels.
Finally SEP is given by the relation

$$P_e = \sum_{m=1}^{M-1} (-1)^{m+1} \frac{(M-1)!}{(M-m-1)!m!} \sum_{m_0+m_1+\dots\dots+m_{L-1}=m} \alpha \times \frac{d^p}{ds^p} \left[(1-2s\sigma_T^2)^{-L} \det\left(I - \frac{2s\sigma_T^2}{1-2s\sigma_T^2} A \right)^{-\beta L} \right]_{s=-m} \quad (16)$$

Where A is $(L \times L)$ matrix and its (k,k) is given by

$A(k,k) = \sqrt{R_g(k,k)}$, β is fading parameter.

Higher order derivative cannot be directly solved easily although a recursive algorithm for this purpose was developed by author [11-13] but here, we derive a simple expression which contains not any functions, derivative and integral and can be solved easily.

4. Closed Form Expression of Symbol Error Probability (SEP)

From eqn. (16), we assume that

$$G(s) = (1 - 2s\sigma_T^2) \det\left(I - \frac{2s\sigma_T^2}{1-2s\sigma_T^2} A \right)^{\beta L} \quad (17)$$

Now,

$$\det\left(I - \frac{2s\sigma_T^2}{1-2s\sigma_T^2} A \right)^{-\beta L} = \prod_{k=1}^{L} \left[\frac{1-2s\sigma_T^2}{1-(1+\lambda_k)2s\sigma_T^2} \right]^{\beta L}$$

Where λ_k are the given values of A

$$\text{So, } G(s) = (1-2s\sigma_T^2)^{-L} \prod_{k=1}^{L} \left[\frac{1-2s\sigma_T^2}{1-(1+\lambda_k)2s\sigma_T^2} \right]^{\beta L} \quad (18)$$

$$\text{Let } \mathrm{H}(\mathrm{s}) = \mathrm{In}\, \mathrm{G}(\mathrm{s})$$

So, P^th derivative of (s) can be written as

$$\frac{d^p}{ds^p} G(s) = G(s) \sum_{\substack{(m_1, m_2, \dots\dots m_p) \\ 0 \le m_1, m_2 \dots\dots m_p \le p \\ m_1 + 2m_{2+\dots\dots+pm_p} = p}} \prod_{k=1}^{p} \frac{k}{m_k!} \left[\frac{1}{k!} \frac{d^k H(s)}{ds^k} \right]^{m_k} \quad (19)$$

and

$$\frac{d^k H(s)}{ds^k} = (-1)^k (2\sigma_T^2)^k (2s\sigma_T^2 - 1)^{-k} L + \sum_{k=1}^{L} \beta \left\{ (-1)^{k-1} (2\sigma_T^2)^k (2s\sigma_T^2 - 1)^{-k} + (-1)^k (2\sigma_T^2)^k (1+\lambda_k)^k \times (2s\sigma_T^2 (1+\lambda_k) - 1) \right\} \quad (20)$$

Substituting eqn. (20) together with (19) and (18) into (16), we get

$$SEP = P_e$$

$$= \sum_{m=1}^{M-1} (-1)^{m+1} \frac{(M-1)!}{(M-m-1)!m!} \sum_{m_0+m_1+\dots\dots+m_{L-1}=m} \alpha \times (1-2s\sigma_T^2)^{-L} \prod_{k=1}^{L} \left[\frac{1-2s\sigma_T^2}{1-(1+\lambda_k)2s\sigma_T^2} \right]^{\beta L}$$

$$\times \sum_{m_1+2m_2+\dots\dots+L-1m_{L-1}=p} \prod_{k=1}^{p} \frac{1}{m_k!k^{m_k-1}} \times \left[(-1)^k \left(2\sigma_T^2\right)^k \left(2s\sigma_T^2-1\right)^{-k} L + \sum_{k=1}^{L} \beta L (-1)^{k-1} \left(2\sigma_T^2\right)^k \left(2s\sigma_T^2-1\right)^{-k} \right.$$
$$\left. +(-1)^k \left(2\sigma_T^2\right)^k \left(1+\lambda_k\right)^k \left(2s\sigma_T^2\left(1+\lambda_k\right)-1\right)^k \}\right]^{m_k} \quad (21)$$

Table 1
M=2, β=2, L=2,
For σ^2_T =0.1, 0.2, 0.5, 1.2

Symbol Error probability (SEP), P_e	**Total SNR γ_T** (dB)
.000001	0
.00001	5
.0001	10
.001	15
.01	20
.1	25
1	30

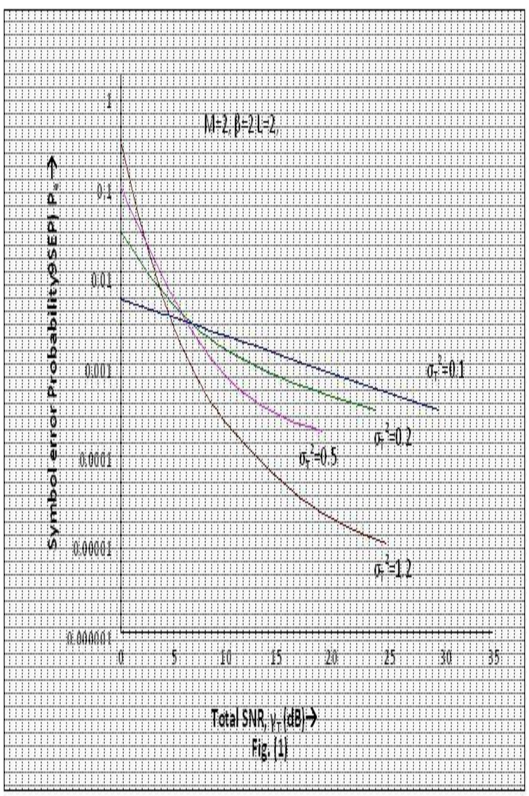

Figure 1, The graph between Symbol Error probability (SEP), P_e and Total SNR γ_T (dB) at M=2, β=2, L=2, for σ^2_T =0.1, 0.2, 0.5, 1.2

Table 2
M=2, β=4, L=1

Symbol Error probability (SEP), P_e	**Total SNR** γ_T (dB)
.000001	0
.00001	5
.0001	10
.001	15
.01	20
.1	25
1	30

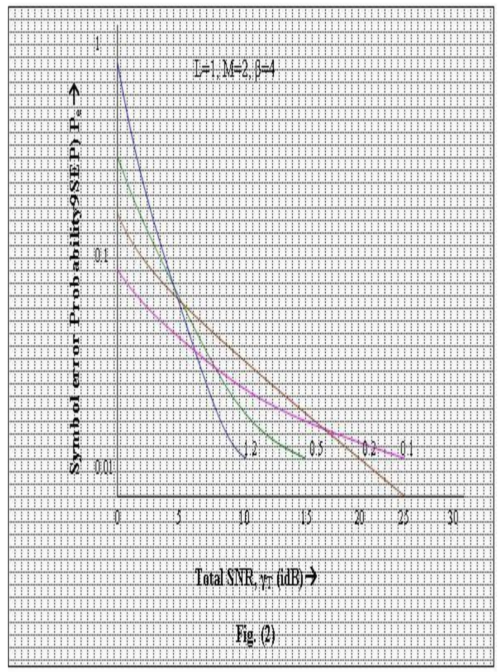

Figure 2, The graph between Symbol Error probability (SEP), P_e and Total SNR γ_T (dB) at M=2, β=4, L=1

Table 3
β=2, L=3
For σ^2_T =0.1, 0.2, 0.5, 1.2

Symbol Error probability (SEP), P_e	**Total SNR** γ_T (dB)
0.0000001	0
0.000001	5
0.00001	10
0.0001	15
0.001	20
0.01	25
0.1	30
1	35

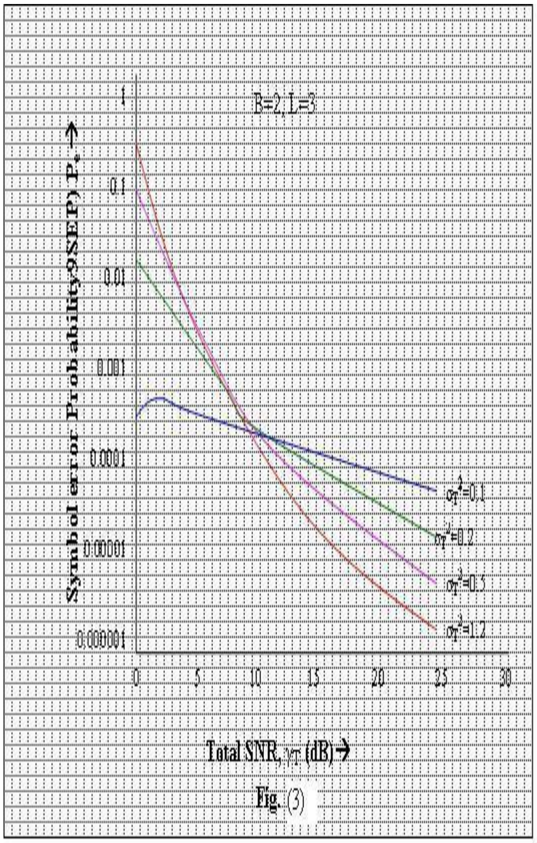

Figure 3, The graph between Symbol Error probability (SEP), P_e and Total SNR γ_T (dB) at β=2, L=3 and for σ^2_T =0.1, 0.2, 0.5, 1.2

Table 4
M=4, β=4, L=4
For σ^2_T =0.1, 0.2, 0.5, 1.2

Symbol Error probability (SEP), P_e	**Total SNR γ_T** (dB)
0.000000001	0
0.00000001	4
0.0000001	8
0.000001	12
0.00001	16
0.0001	20
0.001	24
0.01	28
0.1	32
1	36

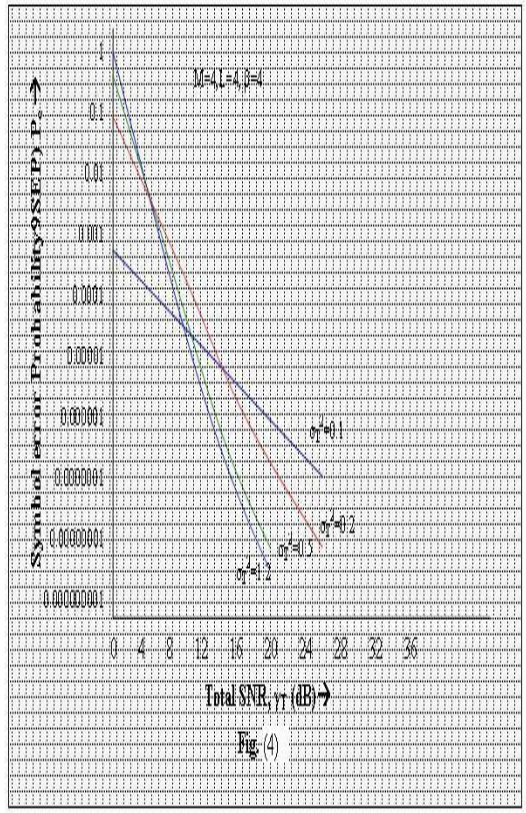

Figure 4, The graph between Symbol Error probability (SEP), P_e and Total SNR γ_T (dB) at M=4, β=4, L=4 and for σ^2_T =0.1, 0.2, 0.5, 1.2

5. Results and Discussion

The experimental value of correlations p for antenna separations d for height h is 1.16 ft and broad side angle $\alpha = 45^0$ have been taken from the empirical curve at 850 MHz Lee [14]. The average SNR $\overline{\gamma_k}$ for k^{th} diversity branch is derived from total SNR γ_T. These are different for all the different branches. Covariance matrix A is related to the array correlation matrix R_g as:

$$A = \frac{\overline{\gamma_k}}{L\beta}\sqrt{R_g} \quad \text{Where} \quad \gamma_T = L\overline{\gamma_k}$$

Probability of error is calculated using eqn. (21) for M=2; L=1, β = 4; L=2, β =2 and σ_T^2 = 0.1, 0.2 0.5, 0.8 and 1 which are plotted in fig. (1) and (2) respectively. The dotted line represents the result of author [13] in the figures. Symbol Error Probability

(SEP) is evaluated for β = 0.5, 1.2, 2, 2.5, 3 and 4 for M=4, L=4 and σ_T^2 =0.5 and depicted in fig (3). The variation of SEP with diversity L taking γ_T as constant for M=4, β =2 and σ_T^2 =0.5 is indicated in fig. (4) Here, therefore from the given figures, we have found the optimum order of diversity for each γ_T . For each γ_T, there is a value of L for which P_e is minimum. Also from the graph, the minimum value of P_e has been achieved at $L \approx 3$ for every γ_T . Thus it can be derived from the above discussions that the average value of SNR per diversity channel is different and depends on the value of M. Here, all the calculations have been performed using MATLAB. In this paper, we have tried to examine the effect of variances σ_T^2 depending on diversity L on symbol error probability, at the combiner output we have considered total SNR$\left(\gamma_T\right)$ and all the calculations have been made with the help of different average SNR $\overline{\gamma_k} = \frac{\overline{\gamma_T}}{L}$ for k^{th} diversity branch.

6. Conclusion

In this paper, now it can be concluded that all in all, the probability of error for NFSK with square law combining for diversity correlated Nakagami channels has been analyzed, where the average SNR per branch is considered to be distinct. The effect of different noise powers for various diversity orders and fading channel parameters on probability of error and unequal SNR distribution is being analysed and studied on which all calculation has been made for total SNR and the average SNR for each diversity branch is achieved from total SNR.[9]. Hence from the all the mentioned figures and graphs, it can be considered that the optimum order of diversity for each γ_T . For each γ_T, there is a value of L for which P_e is minimum. Also from the graph, the minimum value of P_e has been achieved at $L \approx 3$ for every γ_T . Thus it can be derived from the above discussions that the average value of SNR per diversity channel is different and depends on the value of M.

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