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Performance Analysis of Optimum Receiver of Nakagami fading Channel for Non-coherent FSK System

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Abstract

Detection techniques based on the absence of any knowledge of the received carrier phase are referred to as non-coherent detection techniques. The optimum receiver under such a scenario is well known to be a structure that incorporates a form of square-law detection. This demodulation through M matched filters, where each one corresponds to the transmitted base band signals, the decision variables are then formed from the magnitudes of the matched filter outputs and the largest one is selected then. In a multi path environment, it is often difficult in practice to achieve good carrier synchronization. In such instances, it is necessary to employ a modulation for which non-coherent detection is possible. The most popular choice of such a modulation n fading channel applications is orthogonal M-FSK, whose error probability performance in AWGN was considered in the decision variables and the accompanying optimum receiver for non-coherent detection of an energy M-ary signalling set were presented.

Keywords

M-FSK, AWGN, Square Law Detector, Fading Channel.

1. Introduction

In the world of hi-tech, where Detection techniques are based on the absence of any knowledge of the received carrier phase are popularly referred as *non-coherent detection* techniques. Here, the optimum, receivers under such scenarios are well known to be the structure that incorporates a form of square-law detection. This demodulation through M matched filters, where each one corresponds to the transmitted base band signals, the decision variables are then formed from the magnitudes of the matched filter outputs and the largest one is selected then.

In a multipath environment, it is often difficult in practice to achieve good carrier synchronization. In such instances, it is necessary to employ a modulation for which non-coherent detection is possible. The most popular choice of such a modulation n fading channel applications is orthogonal *M*-FSK, whose error probability performance in AWGN is considered in the decision variables and the accompanying optimum receiver for non-coherent detection of an energy *M*-ary signalling set are taken up further. [1, 2]

To proceed the discussion, in this section we will make the much simpler approach that the receiver is designed not to make any attempt at estimating the carrier phase at all. Thus the local carrier used for demodulation is assumed to have an arbitrary phase which, without any loss in generally, can arbitrarily be set to zero. Detection techniques based on the absence of any knowledge of the received carrier phase are referred to as *non-coherent detection* techniques. In mathematical terms, the receiver observes the equivalent base band signal $\tilde{R}(t)\underline{\underline{\Delta}}\tilde{r}(t)e^{-j2\pi fct} = \tilde{S}(t)e^{j\theta c} + \tilde{n}(t)e^{-j2\pi fct,}$ where θ_c is unknown [thus may be assumed to be uniformly distributed in the interval $(-\pi,\pi)$] and attempts to make a decision on $\tilde{S}(t)$. The optimum receiver under such a scenario is well known [5] to be a structure that incorporates a form of square-law detection. Specially, in each symbol interval the receiver first gets complex-conjugate demodulations - the received signal with the zero-phase reference signal (c_r (t) = $e^{j2\pi fct,}$ then passes the result of this demodulation through M matched filters, one each corresponding to the transmitted base band signals.

The decision variables are then formed from the magnitudes (or equivalently, the square of these magnitudes) of the matched filter outputs and the largest one is selected in Fig. (1) In mathematical terms, the decision variables (assuming square-law detection) are given by

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$$z_{nk} = \left|\tilde{y}_{nk}\right|^2 = \left|\int_{nTs}^{(n+1)Ts} \tilde{R}(t)\tilde{S}^{\ *}(t)dt\right|^2, k = 1,2,....,M \qquad (1)$$

Here, $\tilde{S}_k(t), k = 1,2,...,M$, is the set of possible realizations of $\tilde{S}_k(t)$ and the decision is made in favor of the largest of the z_{nk}'s.

Suppose that the modulation was, in fact, M-PSK and one attempted to use the receiver above for detection. Since in the absence of noise the matched filter outputs in the *n*th symbol interval would be given by[$\tilde{N}_n = \int_{nTs}^{(n+1)Ts} \tilde{N}(t)dt$ with the addition of the unknown carrier phase θ_c] $\tilde{y}_{nk} = A_c T_s e^{j(\theta_n - \beta_k)} e^{j\theta_c}, k = 1,2,....,M$, the magnitudes of these outputs would all identical and hence cannot be used for making a decision on the transmitted phase θ_n. We can say in another way, since for M-PSK the information is carried in the phase of the carrier, then since the non-coherent receiver is designed to ignore this phase, it certainly cannot be used to yield a decision on it. In summary, *non-coherent detection cannot be employed with M-PSK modulation.[6]*

Having ruled out M-PSK modulation (which also rule out binary AM because of its equivalence with BPSK), the next logical choice is M-FSK. The matched filter outputs can be seen as:

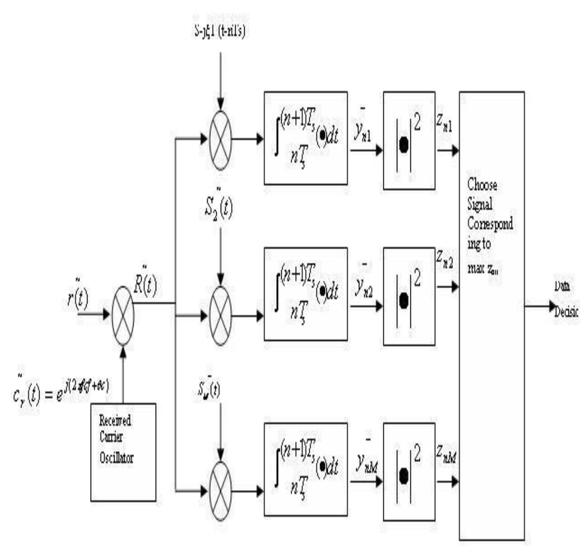

Fig (1) Complex from of optimum for noncoherent detection over the AWGN

Under, Ideal phase coherent conditions, we write these same outputs for the non-coherent case as

$$\tilde{y}_{nk} = A_c e^{j\theta_c} \int_{nTs}^{(n+1)Ts} e^{j2\pi(fn - \xi_k)(t - nT_s)} dt + \tilde{N}_{nk},$$

k= 1, 2, ..., M,

$$\tilde{N}_{nk} = \int_{nTs}^{(n+1)Ts} e^{-j2\pi\xi k(t-nTs)} \tilde{N}(t)dt \qquad (2)$$

Here, now $\tilde{N}(t) = \tilde{n}(t)e^{j2\pi fct}$. Taking the absolute value (or its square) of the $\tilde{y}_{nk}$'s in (2) in the absence of noise removes the unknown carrier phase but leaves the data information, which is now carried in the frequency f_n, unaltered. Thus it is feasible to use non-coherent detection with M-FSK modulation. The additional use of an envelope (or square-law) detector following the matched filters in the non-coherent case will result in a performance penalty relative to the coherent case, where the decision is made based on the matched filter outputs alone. [4-10].

2. Bit Error Probability

Let consider the matched filter outputs described by equation (2) and the assumption of orthogonal signals (corresponding to a minimum frequency spacing $\Delta f_{\min}$= $1/T_s$, which is twice that for coherent detection), the SEP is given by

$$P_s(E) = \sum_{m=1}^{M-1} (-1)^{m+1} \binom{M-1}{m} \frac{1}{m+1} \exp\left[-\frac{m}{m+1}\left(\frac{E_s}{N_0}\right)\right] \qquad (3)$$

and the corresponding BEP is obtained from (3) by the relation

$$P_b(E) = \frac{1}{2}\left(\frac{M}{M-1}\right) P_s(E) \qquad (4)$$

For noncoherent detection of binary FSK, (3) reduces to

$$P_b(E) = \frac{1}{2}\exp\left(-\frac{E_b}{2N_0}\right) \qquad (5)$$

The performance of non-coherent M-FSK is considerably more complicated to evaluate. [7, (5.2.2)] For the binary non-orthogonal case, however, the result can be expressed in terms of the first-order Marcum Q-function as [3]

$$P_b(E) = Q_1(\sqrt{a}, \sqrt{b}) - \frac{1}{2}\exp\left(\frac{a+b}{2}\right) I_0(\sqrt{ab}) \qquad (6)$$

Which is equivalent to $P_b(E) = 1/2[1 - Q_1(\sqrt{b}\sqrt{a}) + Q_1(\sqrt{a}, \sqrt{b})]$ and where

$$a = \frac{E_b}{2N_0}(1 - \sqrt{1-\rho^2}), b = \frac{E_b}{2N_0}(1 + \sqrt{1-\rho^2}) \qquad (7)$$

and ρ is the correlation coefficient of two signals. For ρ=0 (orthogonal signaling), the parameters a and b become a =0 and b=E_0, and using the property of the MArcum Q-function in

$Q_1(0, \beta) = \exp\left(-\frac{\beta^2}{2}\right)$ similarly, $\xi = 0$, in equation

$$Q_1(\alpha, \beta) = Q_1(\alpha, \alpha\xi) = 1 + \frac{1}{2\pi}\int_{-\pi}^{\pi} \frac{\xi^2 + \xi\sin\theta}{1 + 2\xi\sin\theta + \xi^2} \times \exp\left[-\frac{\alpha^2}{2}(1 + 2\xi\sin\theta + \xi^2)\right]$$

$, \alpha \rangle \beta \geq 0, (0 \leq \xi \langle 1)$

We obtain from equation (5).

It is a simple matter now to extend these results to the fading channel.

3. Expression for Symbol Error Probability

Let us consider the equation (3) purely as an exponential of the SNR, then applying the MGF-approach to this equation, we obtain the average SEP:

$$P_s(E) = \sum_{m=1}^{M-1} (-1)^{m+1} \binom{M-1}{m} \frac{1}{m+1} M_{\gamma s}\left(-\frac{m}{m+1}\right) \quad (8)$$

Where the moment generating function M_γ (-s) is obtained from any of the results in equation $I = \int_0^\infty Q(a\sqrt{\gamma})\, p_\gamma(\gamma) d\gamma$ with $\bar{\gamma}$ replaced by the average symbol SNR $\bar{\gamma}_s$. Thus, for Rayleigh fading, using equation $M_\gamma(-s) = \frac{1}{1 + s\bar{\gamma}}, s \rangle 0$

And we have:

$$P_s(E) = \sum_{m=1}^{M-1} (-1)^{m+1} \binom{M-1}{m} \frac{1}{m+1(1+\bar{\gamma}_s)} \qquad (9)$$

which for the special case of binary FSK simplifies to $P_b(E) = 1/(2+\bar{\gamma})$, in agreement with Proakis [8, Eq. (14-3-12)]. For Rician fading,

$$P_s(E) = \sum_{m=1}^{M-1} (-1)^{m+1} \binom{M-1}{m} \frac{1+K}{1+K+m(1+K+\bar{\gamma}_s)}$$

$$\times \exp\left(-\frac{Km\bar{\gamma}_s}{1+K+m(1+K+\bar{\gamma})}\right) \qquad (10)$$

Which agrees with Sun and Reed [4, Eq. (8)], and reduces to eq. (9) when K= 0. Finally, for Nakagami-m-fading, using equation $M_\gamma(-s) = (1 + \frac{s\bar{\gamma}}{m})^{-m}, s \rangle 0$, we obtain

$$P_s(E) = \sum_{l=1}^{M-1} (-1)^{l+1} \binom{M-1}{l} \frac{(l+1)^{m-1}}{[1 + l(1+\bar{\gamma}_2/m)]^m} \qquad (11)$$

Where, we have changed the summation index to avoid confusion with the Nakagami-*m* fading parameter. As expected, eq. (11) reduces to eq. (9) when *m* =1. The results for average BEP over a fading channel can be obtained, for the AWGN channel, by applying the relation between bit and symbol error probability in equation $P_b(E) = \frac{1}{2}(\frac{M}{M-1})P_s(E)$ to the results. Furthermore, we have specifically addressed scheme transmitted over a slow, flat fading channel and detected non-coherently at the receiver.

For non-orhtogonal *M*-FSK, we observed that a simple analytical result for average BEP over the AWGN is possible for the binary case, namely eq. (11) to extend this result to the fading channel:

$$P_b(E) = \frac{1}{2}[1 - Q_1(\sqrt{b}, \sqrt{a}) + Q_1(\sqrt{a}, \sqrt{b})] \quad (12)$$

and then make use of the alternative representation of the Marcum Q-function in to allow application of the MGF-based approach . Using the definitions of *a* and *b* in eq. (12), the result of this application produces: [9]

$$P_b(E) = \frac{1}{4\pi}\int_{-\pi}^{\pi} \frac{1-\xi^2}{1+2\xi\sin\theta+\xi^2}$$

$$\times M_\gamma\left(-\frac{1}{4}(1+\sqrt{1-\rho^2})(1+2\xi\sin\theta+\xi^2)\right)d\theta$$

$$\xi \underline{\Delta} \sqrt{\frac{1-\sqrt{1-\rho^2}}{1+\sqrt{1-\rho^2}}} \qquad (13)$$

Here, ρ is the correlation coefficient of the two signals.

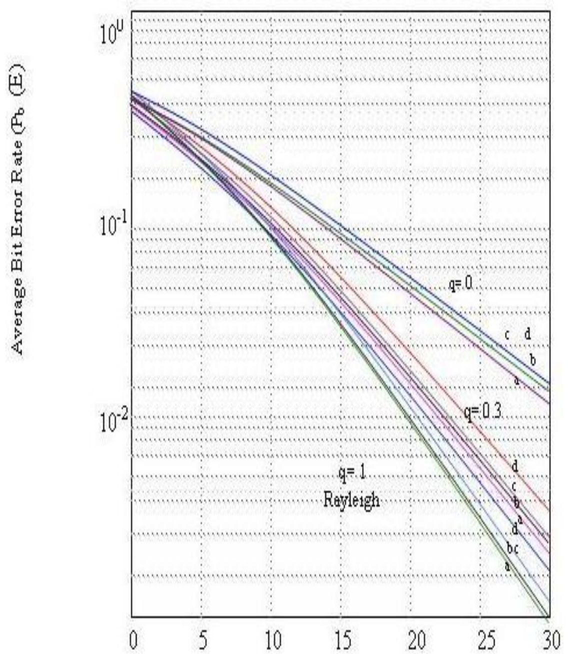

Fig (2) Average Signal to Noise Ratio per Bit (dB) Average BEP of correlated BFSK over a Nakagani-q fading channel

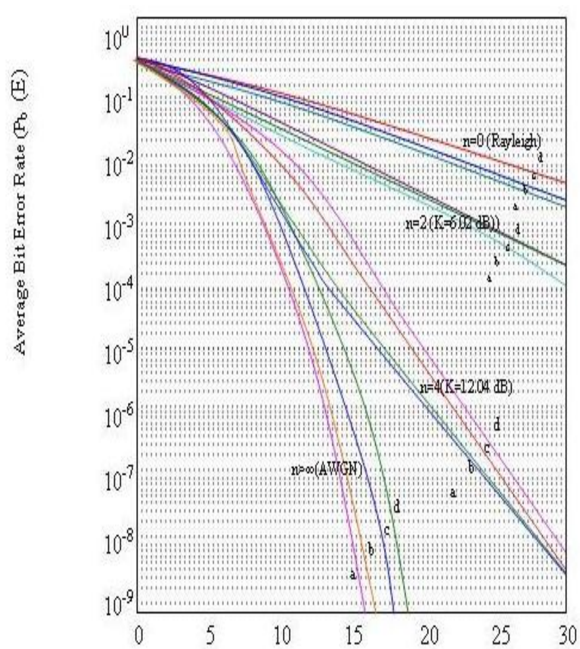

Fig.3a. Average signal to Noise Ratio per Bit (dB) Average BEP of conelated BFSK over a Nakagami –n channel

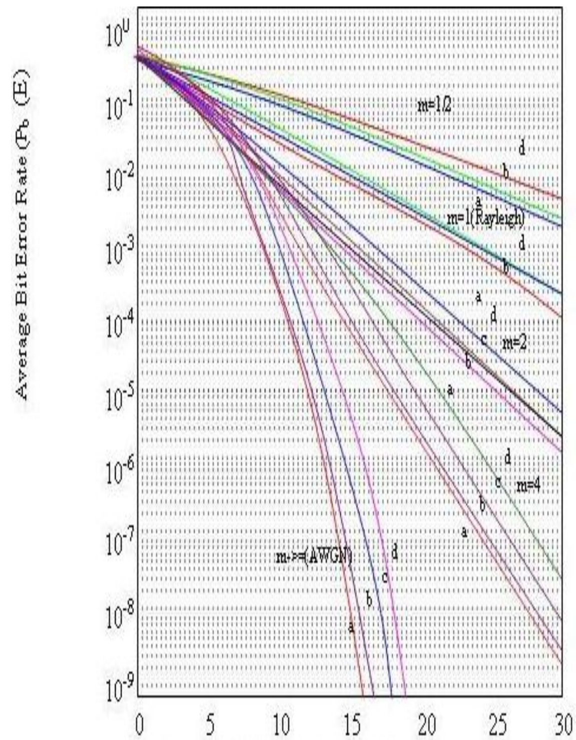

Fig.3b Average Signal to Noise Ratio per Bit(dB) Average BEP of conelated BFSK over a Nakagami-mchannel

4. Conclusion

Thus, we can conclude from the above discussions and derivations made, that we have obtained specific results for the various fading channels, one merely substitutes the appropriate MGF from in equation (13), analogous to what done previously for the orthogonal signalling case. The specific analytical results are left as an exercise for the reader. After making the aforementioned substitutions and the derivation of the numerical results which are obtained from eq. (13); Fig. (2) and Fig (3a & b) illustrate the average BEP performance for Nakagami-*n* (Rice), and Nakagami-*m* channels.

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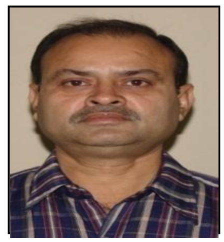

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