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# Compressed Sensing Reconstruction of an Audio signal using OMP

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## Abstract

***Compressive sensing (CS) is an evolving technique for data acquisition that promises sampling a sparse signal from a far fewer measurements than its dimension. Compressive sensing enables a potentially large reduction in the sampling and computation cost for sensing signals that have sparse representation. The signal having sparse representation can be recovered from small set of linear, non-adaptive measurements. This papermainly focuses on fixing threshold value for proper reconstruction of an audio signal. An audio signal is better reconstructed for the threshold value between (-0.02 to +0.02). Various performance parameters are measured which describe exact reconstruction of the signal. For proper reconstruction Orthogonal Matching Pursuit algorithm is used.***

## Keywords

***Compressive sensing, sparsity, measurement matrix and OMP.***

## 1. Introduction

Compressive sensing (CS) is an emerging and revolutionary technology where the signal of interest is efficiently acquired from very few non-zero coefficients which mean CS strongly relies on sparsity of the signal.CS is a technique that allows going beyond the Nyquist-Shannon sampling.It is very simple and efficient technique that provides both sampling and compression along with the encryption of data.[1][2][3]. The theory of compressive sensing was first developed by Candes at el and Donoho in 2004.

Most of the signal acquisition systems have been designed based on the Nyquist Shannon sampling theorem. According to sampling theorem, an encoded signal can be reconstructed exactly if it is sampled at a frequency called sampling frequency that is at least twice the maximum frequency present in the signal. The main drawback of Nyquist sampling theorem is that it leads to large number of samples and most of them are probably not required for its reconstruction. Also in some systems, increasing the sampling rate beyond certain point make them expensive [4]. The importance of CS is that it helps in sampling the signal below Nyquist rate. Hence CS is essential as far as storage and transmission are concerned. It also allows reconstruction of the signal at a frequency much less than the sampling frequency. Hence number of samples required for reconstructing the signal would decrease. Reducing the number of measurements will reduce the time and cost of signal acquisition. CS has applications in many fields. The prominent examples include magnetic resonance imaging (MRI), image acquisition, wireless communication and radar [6].

The paper is organised as follows: section 2 provides background of compressive sensing, section 3 provides reconstruction algorithms, section 4 provides performance metrics, section 5 provides simulation results and finally section 6 provides conclusion.

## 2. Background

### 2.1 Sparse approximation:

A signal $f \in R^N$ is $T$-sparse in a basis $\Psi$ if, there exist a vector $x$ with $\|x\|_0 = T$ (where $T$ is no of non-zero coefficients of $x$) such that $f = \Psi x$. Signal compression mainly relies on a known basis $\Psi$ such that for a signal of interest $f$ there exit a $T$-*sparse approximation* $f_T$ in $\Psi$ that yields small approximation error $\|f - f_T\|_2$. $T$-*sparse approximation* of $f$ is found through hard

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thresholding. Here only $T$ largest coefficients of $x$ are preserved and rest other coefficients are set to zero.

### 2.2 Compressive sensing:

Compressive sensing is an efficient acquisition method for signals that are sparse in a basis $\Psi$. In CS we measure the inner products of the signal against the set of measurement vector$\{\varphi_1, \varphi_2, \ldots, \varphi_M\}$. By taking measurement vectors as rows of the measurement matrix $\Phi \in R^{M\times N}$ the procedure can be written as $y = \Phi f = \Phi\Psi x$ with $y \in R^M$ containing CS measurements. Now we aim to recover the signal $f$ from fewest possible measurements $y$, for this we employ standard sparse approximation algorithm to recover signal $x$ by finding a sparse approximation of $y$ using the basis $A = \Phi\Psi$. Fig.1 briefly describes the concept of compressive sensing reconstruction of signal.

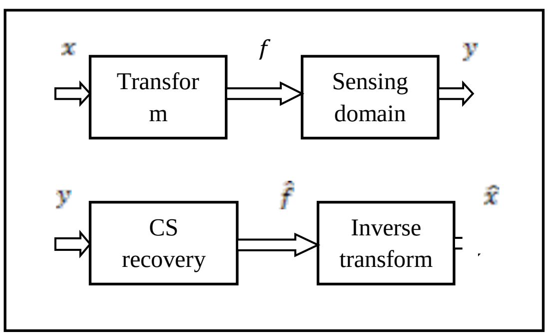


**Fig.1: Block diagram of CS**

#### 2.2.1 Mathematical model:

Let $x \in R^N$ be a speech signal and let $\Psi = \{\psi_1, \psi_2, \ldots, \psi_N\}$ be a basis vectors spanning$R^N$. The signal is said to be sparse if,

$$f_T(t) = \sum_{i=1}^{N} x_{T_i}\, \psi_i(t) t = 1,2, \ldots, N \qquad (1)$$

Where $f_T$ is a sparse vector with only $T$ non-zero elements where$T \ll N$. Based on CS theory, sample the signal $f$ in the sampling domain $\phi = \{\varphi_1, \varphi_2, \ldots, \varphi_N\}$ and reconstruct the signal at the receiver with the full knowledge of random bases. The measurement signal is defined as

$$y(k) = \sum_{i=1}^{N} f_i\, \varphi_i(k) k = 1,2, \ldots, M \qquad (2)$$

Where $\Phi$is a measurement matrix of dimension $(M \times N)$.The vector $y$ can be reconstructed perfectly if $M > T \log N$ measurements which means to reconstruct the sparse signal perfectly, the measurement matrix has to satisfy two important properties namely, restricted isometry property (RIP) and property of incoherence.

##### 2.2.1.1 Restricted Isometry Property (RIP):

For any arbitrary constant $\delta$, measurement matrix should satisfy the following condition.

$$1 - \delta_T \|x\|_2^2 \le \|Ax\|_2^2 \le 1 + \delta_T \|x\|_2^2 \qquad (3)$$

Where, $0 < \delta < 1$

A matrix has RIP if $\delta_T > 0$. Calculating $\delta_T$ for a given matrix takes high computation; hence random matrices have been used as a measurement matrix.

##### 2.2.1.2 Incoherence condition:

This condition states that, the rows of measurement matrix $\Phi$ should be incoherent with the columns of basis vector$\boldsymbol{\Psi}$. If they are not incoherent then the product of these two will give an identity matrix and this will fail the compressive sensing technique.

Checking whether the matrix satisfies the RIP condition and condition of incoherence is complex procedure. Hence randomly generated matrices have been used as a measurement matrix.

### 2.3 Frequency-sparse signals:

In order to exploit sparsity in CS, we require discrete representation of the signal. Hence the signal has to be converted into a frequency domain to obtain discrete representation of the signal. Here Discrete Cosine Transform (DCT) has been used as a tool of choice for frequency-sparse signals.

#### 2.3.1 Discrete Cosine Transform (DCT):

DCT will remove the redundancy between the neighbouring values which will result in uncorrelated transform coefficients. These coefficients then can be encoded independently. In DCT the energy content is concentrated more on the lower order coefficients. All the spectral components are purely real in it. We can increase the sparsity of the signal by thresholding the low value components without affecting the signal quality.[7]

## 3. Reconstruction Algorithms

Signal reconstruction plays a very important role in compressive sensing theory where the signal having sparse representation can be reconstructed using very few measurements. Sparse signal can be reconstructed using different optimization techniques. The different optimization techniques include

### 3.1 L1-Minimization:

In CS, reconstruction using *L1-minimization* is an efficient approach. Here *L1* is nothing but an *L1* norm. *L1-minimization* technique solves the following convex optimization problem,

$$min \parallel f \parallel_1 \ subject\ to\ y = \Phi f \tag{4}$$

Solution to the above equation would give a reconstructed signal.

### 3.2 Greedy approach:

Greedy algorithm mainly relies on iterative approximation of signal coefficients and support. The algorithm computes the support of sparse signal $f$iteratively. Once the support of the signal is computed correctly, then the pseudo-inverse of the measurement matrix can be used to reconstruct the signal$f$, Main advantage of using this approach is for its speed of computation. Orthogonal matching pursuit (OMP) is one of the greedy algorithms used here.

#### 3.2.1 Orthogonal Matching Pursuit (OMP):

OMP is a greedy reconstruction algorithm which can be formulated as

$$min \parallel y - \Phi\Psi x \parallel_1 \ subject\ to\ y = \Phi f \tag{5}$$

The idea behind this algorithm is to find the best column of the measurement matrix $\Phi$ which contributes to the observation vector $y$. At each iteration, column of $\Phi$ is chosen such that it is strongly correlated with the observation vector $y$.Then the contribution of that column is subtracted from $y$ and the same procedure is repeated for the residual of $y$. After $T$ iterations, the algorithm would have identified the correct set of columns. The residual at the end is nothing but a reconstructed signal $f$ [5].

**Algorithm: Orthogonal matching pursuit (OMP)**

**Inputs:** measurements $\boldsymbol{y}$, measurement matrix $\boldsymbol{\Phi}$ and sparsity $\boldsymbol{T}$.
**Output:** sparse reconstruction $\boldsymbol{f^k}$
**Initialize:** count $\boldsymbol{k = 0}$; residual $\boldsymbol{r^0 = y}$; support$\boldsymbol{S^0} = \emptyset$.
While $\boldsymbol{k < T}$

$$\boldsymbol{k = k + 1}$$

(Identify) $\boldsymbol{s^k = argmax_j \left|\langle r^{k-1}, \Phi_j \rangle\right|}$
(Augment) $\boldsymbol{S^k = S^{k-1} \cup s^k}$
(Estimate) $\boldsymbol{f^k = argmin_f \left\| r^{k-1} - \Phi_{S^k} f \right\|_2}$
(Update) $\boldsymbol{r^k = r^{k-1} - \Phi_{S^k} f}$
End

## 4. Performance Metrics

To evaluate the performance of the proposed compression scheme, several objective tests were made. Factors such as mean square error (MSR), signal to noise ratio (SNR) and perceptual evaluation speech quality (PESQ) were taken into considerations to measure the performance of the reconstructed signal [8].

### 4.1 Mean square error (MSE):

The Mean Square Error is defined as the amount of difference between the actual signal and the reconstructed signal. It is given by

$$MSR = \frac{\sum_n (x[n] - y[n])^2}{n} \tag{6}$$

where $x[n]$is an original speech signal, $y[n]$ is reconstructed signal and $n$ is the length of the signal.

### 4.2 Signal to noise ratio (SNR):

Signal to Noise Ratio is defined as the ratio of signal power to the noise power, Higher the SNR, better is the signal quality. It is measured in decibel (db). The signal to noise ratio is defined by

$$SNR = 10 \log_{10} \left[\frac{\sum_n x[n]^2}{\sum_n (x[n] - y[n])^2}\right] \tag{7}$$

where$x[n]$ is an original speech signal, $y[n]$ is reconstructed signal and $n$ is the length of the signal.

### 4.3 Perceptual evaluation speech quality (PESQ):

The Perceptual Evaluation of Speech Quality is an objective method of measuring speech quality. It is calculated using subjective Mean Opinion Scores (MOS). The range of PESQ lies within 0 to 4.5. Here the lower values are interpreted as poor speech quality and higher values are interpreted as better speech quality [9].

## 5. Simulation Results

The experiment is conducted on an audio file “HI” which contains 16000 samples which mean the length of the audio file is of 16000. This test is conducted using MATLAB. The whole signal is divided into 4 frames. Each frame contains 4000 samples. Compressive sensing is applied for each of the frames using random measurement matrix. The signals of each frame are reconstructed using orthogonal matching pursuit (OMP) algorithm. Each reconstructed frame is combined to obtain complete reconstructed signal. Fig.2 shows the recorded speech signal of length 16000 which is to be compressed using DCT and random measurement matrix.

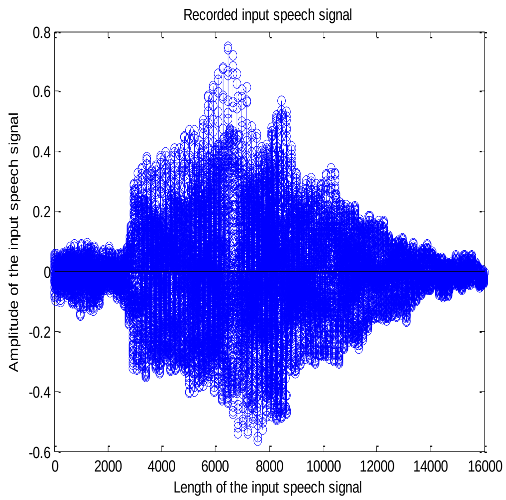


**Fig. 2: Recorded signal**

Fig.3 shows the reconstructed spectrum at the receiver side which is obtained using OMP reconstruction algorithm

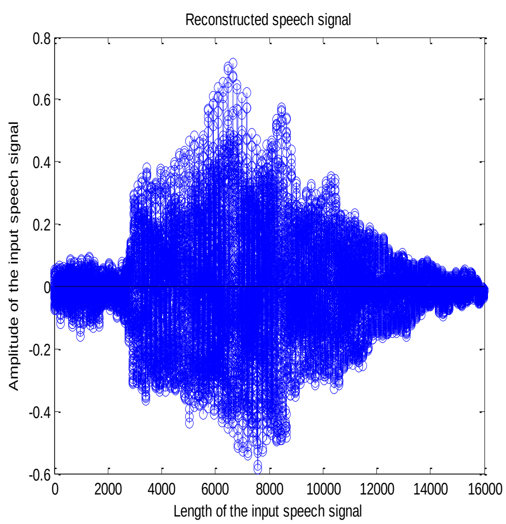


**Fig.3: Reconstructed signal**

**Table1: Compression result of signal "HI"**

| Length of signal N | Threshold window UL | Threshold window LL | MSE | SNR | PESQ |
|---|---|---|---|---|---|
| 16000 | 0.02 | -0.02 | 1.72E-4 | 57.07 | 3.9 |
| 16000 | 0.04 | -0.04 | 4.608E-4 | 53.14 | 3.2 |
| 16000 | 0.04 | -0.06 | 4.851E-4 | 53.23 | 2.5 |
| 16000 | 0.06 | -0.06 | 7.34E-4 | 51.35 | 1.9 |

Table 1 shows how the signals performance will vary with the variation in the threshold window. From the table it is observed that, as the threshold value increases the mean square error (MSE) between the actual and the reconstructed signal increases and the signal to noise ratio (SNR) and perceptual evaluation speech quality (PESQ) of the signal gets decreases. From this it is evident that the signal can be reconstructed perfectly if the threshold value is very less.

## 6. Conclusion

Compressive sensing can be efficiently used in speech processing technique as it will reduce the storage capacity and increase the data rate of the signal. CS technique can be used as a substitute for the traditional Nyquist Shannon Sampling as it uses less than half of the measurements to reconstruct the signal. Experiments were carried out for various threshold windows. For the threshold window of "-0.02 to +0.02", better reconstruction of audio signal is possible. Various performance parameters are measured which describe exact reconstruction of the signal. For proper reconstruction Orthogonal Matching Pursuit algorithm is used.

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