

Modelling of cantilever subjected to air-blast

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Abstract

This paper considers the response of cantilevers subjected to air-blast. The response of cantilevers (e.g. lamp-posts, street signs) had been observed and this was used to assess the effective energy yield of the two nuclear weapons dropped at Hiroshima and Nagasaki in 1945. Since then, a number of other researchers have also considered the behaviour of simple cantilevers of various cross-sections to monitor the blast parameters. The cantilevers considered in this paper are circular in cross-section and are assumed to have a ductile failure mode. It is observed that after being loaded the ductile cantilevers were straight throughout the entire length except at the clamped end where the cantilever was bent through an angle θ . Thus for the cantilever subjected to blast pressure, the critical parameter was the angle of deformation at the base. The tool used for simulation of cantilever response is AUTODYN- 3D (version 4.3). This software is specifically suitable for analysis of non-linear dynamics problems. In AUTODYN state-of-the-art analysis coupled with modern graphics provide appropriate environment for solving complicated engineering problems. Thus AUTODYN is an engineering and scientific tool for solving complex problems. AUTODYN is the right tool for analysis of nonlinear dynamic problems, which includes deflection of cantilever, and variation of material properties. This software has special features which enable the dynamic analysis of a cantilever subjected to varying pressure with time. Also AUTODYN has options to include strain-rate effects in the analysis of the problem. In special cases user defined subroutines can be incorporated in AUTODYN. AUTODYN has multi-processor system. For material modelling different processors namely, Euler, Lagrange, Arbitrary Lagrange Euler (ALE), Shell and Beam are available, which may be chosen corresponding to type of material. In this paper twenty cantilever specimens of 3mm and 5mm diameter and of length ranging from 50 mm to 700

mm were considered. These cantilever specimens were subjected to pressure of 50 kPa. Cowper-Symonds strain rate model is used for the analysis as used strain-rate is $2.72 \times 10^6 \text{ s}^{-1}$. Details of the results obtained from numerical simulation using AUTODYN are presented in this paper. These results are also compared with earlier experimental results obtained by the author at Pulau-Senang.

Keywords

cantilever, dynamic, ductile failure, air-blast, AUTODYN, nonlinear.

1. Introduction

The devastating events that occurred at Hiroshima and Nagasaki in 1945, Lord Penny[1] determined the effective energy yields of the two explosions. The weapons yields were determined from observation of damage to various simple structures such as bent poles, toppled grave stones, crushed paint cans, broken glass windows, dished-shaped cabinet walls in the vicinity of the explosion. The various modes of failure of these simple devices were used to quantify properties of the blast wave. Consequently, Ewing and Hanna[2] and Baker, *et. al.*[3] developed cantilever gauges with rectangular cross-sections and calibrated these gauges in the impulsively loaded regime. Later, Dewey and McMillan[4] studied the effect of explosions on the deformation of solder cantilevers to determine the uniformity and efficiency of these events. Smith and Hetherington[5] extended further, the study carried out by Baker, of the energy approaches for beams of rectangular cross-section without geometric non-linearity. More recently van Netten[6] investigated the influence of blast wave on simple cantilevers.

The various modes of failure of these simple devices were used to quantify properties of the blast wave. Thus it is decided to consider cantilever for modelling. Recently, Mark C. Price *et.al* [7] in 2011 used AUTODYN as a tool to model aluminium foil subjected to impact. During the modelling it is observed that, every AUTODYN problem follows a

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few basic steps for setting up an analysis. For analysing the problem with the use of AUTODYN the user has to provide the information related with the problem. For all types of problems the following five major categories of information must be supplied to the program,

1. Geometry, 2. Materials, 3. Initial conditions, 4. Boundary conditions 5. Editing

2. Cantilever modelling

AUTODYN utilizes the differential equations governing unsteady material dynamic motion, to express the local conservation of mass, momentum and energy. In order to obtain a complete solution, in addition to appropriate initial and boundary conditions, it is necessary to define a relation between the flow variables. This can be done by a material model which relates stress to deformation and internal energy.

2.1 Material modelling with AUTODYN

The material model to be chosen will depend on the physical properties of the material in the problem. For all type of problems the following four major categories of information must be supplied to the program

1. Equation of state (EOS): There are large numbers of EOS available in AUTODYN. For cantilever modelling, 'Linear EOS' is chosen. This EOS is best suitable corresponding to the materials involved in this problem. In this EOS a bulk modulus and reference density is defined.
2. Yield model: There are large numbers of Yield models available in AUTODYN. A yield model having strain-rate effect based on Cowper-Symonds relationship is not available in AUTODYN yield model library. Thus for a cantilever modelling 'User defined yield model' is chosen. This 'User defined yield model' uses subroutine EXYLD linked into the standard AUTODYN file. Both the yield surface and shear modulus needs to be defined.
3. Failure models: There are large numbers of Failure models available in AUTODYN. The Failure model chosen for cantilever modelling is 'None', which assumes that the material will never fail during the analysis.

This failure model enables the analysis to continue while the material passes from elastic to plastic state. The analysis continues till the velocity at all the nodes attains zero value.

4. Erosion models: Erosion is initiated for an element when the specified strain limit is reached. The element is transformed into a free mass node (retained inertia) or discarded (no retained inertia). Erosion is applicable to materials contained within Lagrange, ALE and Shell sub-grids. It is not applicable to materials contained in Eulerian sub-grids. The Erosion model chosen for cantilever modelling is 'None', which assumes that the material will never undergo erosion during the analysis as it will never attain the specified strain limit.

2.2 Beam processor

For modelling a cantilever, the beam processor is used. The beam processor represents the deformation of a cantilever more accurately as compared with Lagrange and Euler processor because beam processor divides the given structure in number of small segments which results in accurate calculation of deformations. Also beam processor is having facility of applying force at various node points.

3. Cowper-Symonds Model

For hyper strain rate effects, Marc C. Price *et.al.*[8] used Preston-Tonks-Wallace strength model, as the strain rate presumed during the analysis was in the range of 10^{11} s^{-1} . But the strain rate in current problem is in the range of $2.72 \times 10^6 \text{ s}^{-1}$ [9,10], thus a Cowper-Symonds model is chosen by the author for the modelling.

The Cowper-Symonds relation represents a rigid-perfectly plastic material with dynamic yield or flow stress $\sigma(\dot{\epsilon})$ which depends on the strain-rate $\dot{\epsilon}$. Thus

the ratio of dynamic to static yield stress $\frac{\sigma_{dy}}{\sigma_y}$ is as

given by, the relationship which represents a rigid-plastic material with dynamic yield or flow stress $\sigma(\dot{\epsilon})$ that depends on the strain-rate $\dot{\epsilon}$. The ratio of dynamic (σ_{dy}) to static yield strength (σ_y) is[11],

$$\frac{\sigma_{dy}}{\sigma_y} = 1 + \left(\frac{\dot{\epsilon}}{\dot{\epsilon}_0} \right)^{1/s} \quad \text{In AUTODYN the}$$

Cowper-Symonds model is not available. Thus for a cantilever modelling 'User defined yield model' is chosen. This 'User defined yield model' uses subroutine EXYLD linked into the standard AUTODYN file. A small subroutine is written in FORTRAN to solve Cowper-Symonds equation.

For Aluminium 6061-T6[10], substituting the constants obtained by Symonds, $s = 4$ and $\dot{\epsilon}_0 = 2.72 \times 10^6$ /second, this model can be used.

4. Step-by-step procedure of formulation

The step-by-step procedure of formulation of a problem is illustrated for aluminium and copper specimens as detailed below.

4.1 Example of aluminium cantilever

Consider the case of 5mm solid circular aluminium rod of 550 mm unsupported length (Fig. 1). For illustration, obtain the deflection of the cantilever rod when it is subjected to a peak dynamic pressure of 50 kPa at 37.4 ms.

Solution:

Group- Aluminium

Object type- Single beam

No. of elements- 10

In AUTODYN on starting menu screen choose "Create" option.

a. Create- In "Create" first assign identification number to a file.

Ident- B10550

In the identification number chosen for this file B stands for beam, last three digits indicate the length of the specimen.

Heading- Interaction of aluminium cantilever

Sub-grid- Beam;

Processor- Beam

b. Zoning-

In zoning X, Y, Z coordinates of first and last point of cantilever are specified.

Line-

X coordinate of start point = 0

Y coordinate of start point = 0

Z coordinate of start point = 0

X coordinate of end point = 0

$$Y \text{ coordinates of end point} = \frac{10}{9} \times L =$$

$$\frac{10}{9} \times 550 = 611.11 \text{ mm}$$

Z coordinate of end point = 0

Geometric ratio = 1.0

Yield stress (σ_y) - 205.00 MPa

Hardening constant- 0; Hardening exponent- 0

Strain-rate constant- 4; Thermal softening exponent- 0; Melting temperature- 933.47 °K

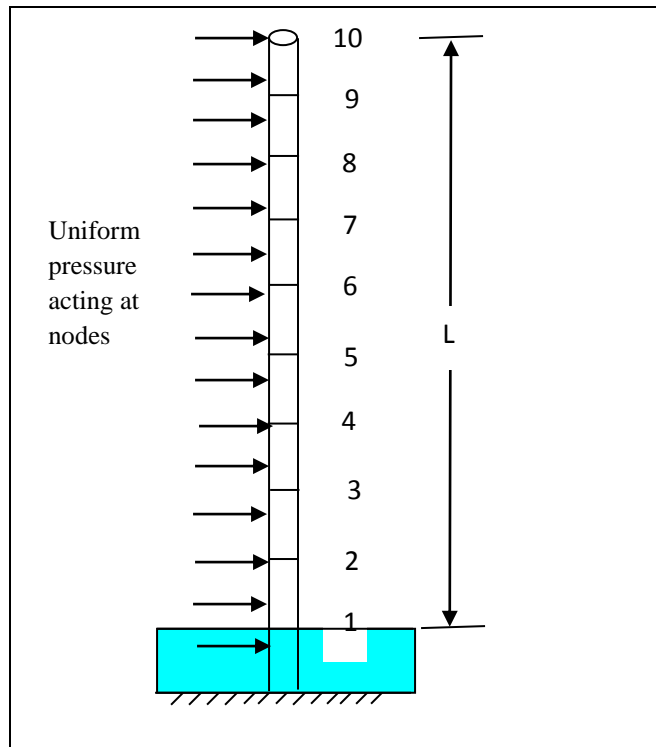


Fig.1: Cantilever rod subjected to pressure

c. Fill- Aluminium

Name of cross-section- Cylinder

Type- Circular

Outer radius- 2.5 mm

Boundary condition- Node 1 and node 2: Fix-X, Fix-Y, Fix-Z

Node 1 to node 11: force- constant = $50 \times \pi \times d$ kPa

Global-

add or modify the material

Material- Aluminium

E.O.S.- Linear

Strength model- User defined

Failure model- none
 Erosion model- none
 Reference density- 2700 kg/m^3
 Bulk modulus (C) - $9.237 \times 10^4 \text{ MPa}$
 Shear modulus (G) - $3.08 \times 10^4 \text{ MPa}$

Then the problem is executed. The results are saved after every 1000 cycles and continued the process till the velocity at each node is zero.

The slides in Fig. 2 to Fig. 4 show the deflection of aluminium cantilever rod after every 5000 cycles. After 10,000 cycles it may be observed that the velocity at each node is zero (Fig. 5). Therefore, after 10000 cycles the problem was terminated.

To obtain the deflection corresponding to 10,000 cycles, post processor is used. Since the integral of velocity is deflection, the option integrate is used. Fig. 6 shows the deflection of node 10.

If this procedure is repeated for various lengths of aluminium cantilevers a set of length and corresponding deflection may be obtained. A few such cases are evaluated and tabulated in Table 1.

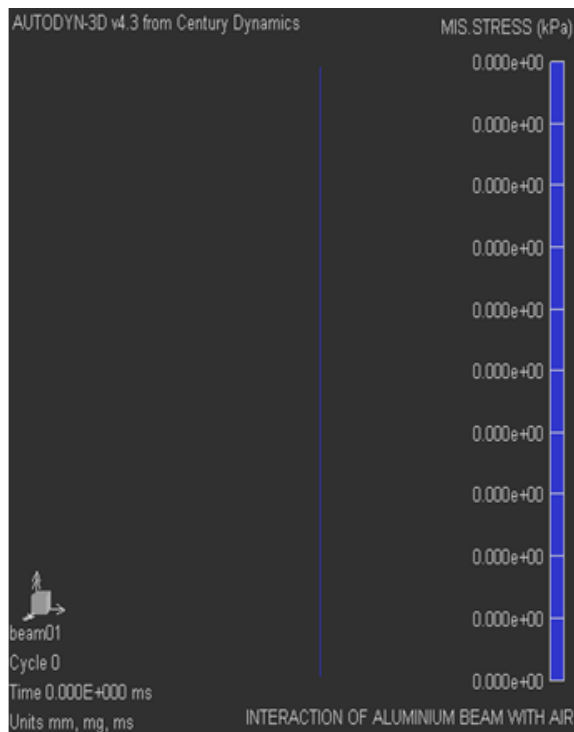


Fig. 2: Initial position of aluminium cantilever corresponding to zero cycle

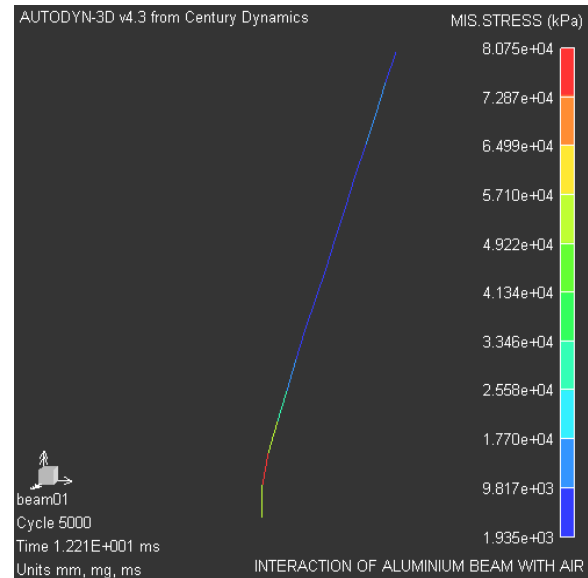


Fig. 3: Response of aluminium cantilever at 5000 cycles

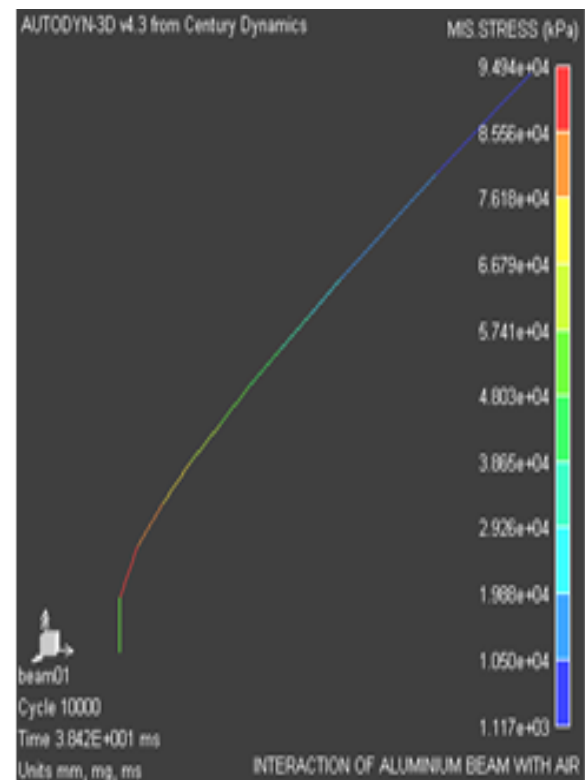


Fig. 4: Response of aluminium cantilever at 10000 cycles

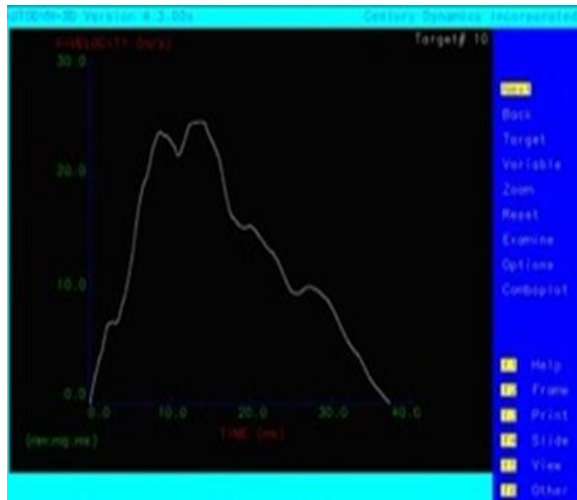


Fig.5: Velocity-time curve of node no. 10- aluminium cantilever at 10,000 cycles

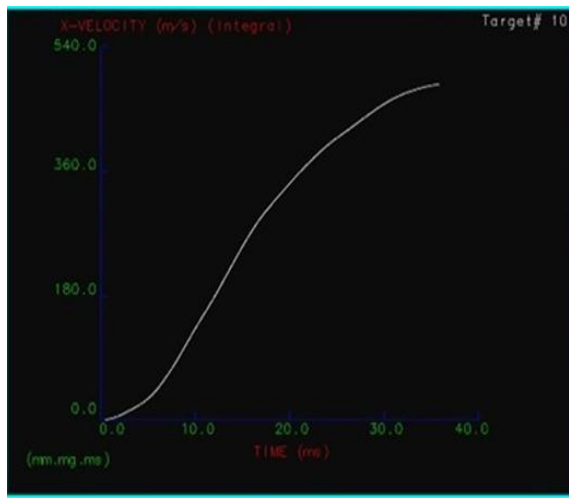


Fig.6: Deflection-time curve of node no. 10- aluminium cantilever at 10,000 cycles

Table. 1: Aluminium cantilever subjected to a peak dynamic pressure of 50 kPa (Numerical solution)

(a) 5mm diameter aluminium bars

| | Ø | L | θ° | | Ø | L | θ° |
|---|---|-----|----------------|---|---|-----|----------------|
| a | 5 | 250 | 57.82 | f | 5 | 500 | 194.57 |
| b | 5 | 300 | 81.27 | g | 5 | 550 | 228.97 |
| c | 5 | 350 | 109.01 | h | 5 | 600 | 260.14 |
| d | 5 | 400 | 137.86 | i | 5 | 650 | 295.67 |
| e | 5 | 450 | 162.33 | j | 5 | 700 | 326.70 |

(b) 3mm diameter aluminium bars

| | Ø | L | θ° | | Ø | L | θ° |
|---|---|-----|----------------|---|---|-----|----------------|
| a | 3 | 50 | 9.01 | f | 3 | 175 | 83.48 |
| b | 3 | 75 | 23.46 | g | 3 | 200 | 99.00 |
| c | 3 | 100 | 37.86 | h | 3 | 225 | 115.62 |
| d | 3 | 125 | 53.49 | i | 3 | 250 | 132.38 |
| e | 3 | 150 | 69.02 | j | 3 | 275 | 149.03 |

The variation of angle of deflection corresponding to required length may also be represented graphically as in Fig. 7.

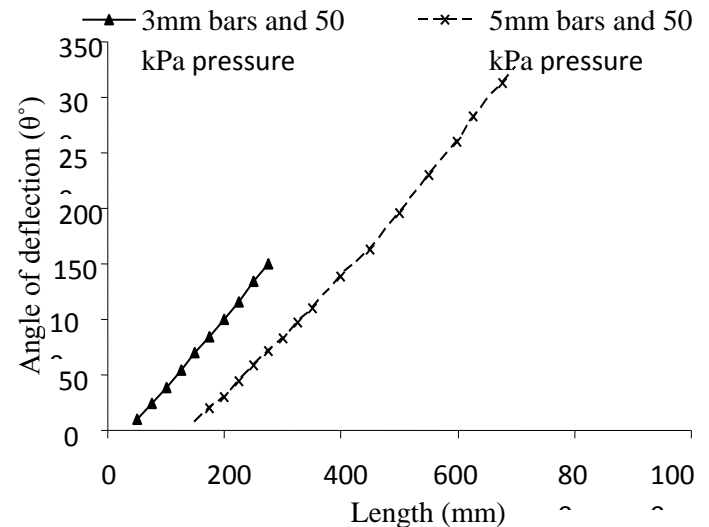


Fig. 7: Variation of angle of deflection with length for aluminium cantilevers- Numerical solution

5. Discussion

Cantilevers is a basic tool which may be used for assessing effects of air-blast on structures. The response of canrilever help the researcher to evaluate possible divastating effect of the explosion on the structure within its range.

5.1 Importance of AUTODYN for cantilever simulation

Field testing of structures subjected to air-blast involves large number of difficulties-

- The cost of explosion test is enormously large
- The test involves many uncertainties such as debris throw, failure of monitoring devices due to sudden rise in temperature and pressure

AUTODYN simulation provides result more accurately and with negligible cost as compared to field test.

5.2 Comparative analysis of AUTODYN modelling results

A comparative study of cantilever modelling using conventional methods along with experimental test at Pulau-Senang was carried out by Kulkarni and Lok[12]. The results obtained through both the methods matches well within the limits with the results obtained by the author by using AUTODYN simulation.

6. Conclusion

The response of uniform solid circular cantilevers subjected to small magnitude of air-blast loading has been presented. Using software AUTODYN numerical analysis of cantilever subjected to dynamic load is carried out. The results obtained for aluminium cantilever subjected 50 kPa are presented in Table 1.

However a number of effects such the influence of variation of heat on the material, pressure-time variation was not considered. The technique could be used by engineers to estimate weapons yield from accidental or deliberately-initiated explosions from the deflections of uniform cantilever structures.

From Fig. 7, it may be observed that the variation of length with angle of deflection is almost linear. The proposed simple AUTODYN model is relatively accurate in predicting the response. However, in actual field explosion, several lengths must be incorporated to ensure that deflections are captured. Cantilevers located closest to the explosion source have a greater discrepancy that those further away. This could be due to the more uniform pressure over a short duration at longer distance from GZ.

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Nomenclature

| | |
|----------------|---|
| d | Diameter of section |
| E | Young's modulus of elasticity |
| F | Transverse force |
| F(t) | Shear force on the section adjacent to the colliding mass |
| f(t) | Non dimensional force acting on mass m |
| F _c | Static plastic collapse force |
| I | Second moment of area |
| L | Length |
| M | Bending moment |
| M _r | Elastic resisting moment |
| M _y | Yield Moment |

| | |
|--|--|
| t | Time |
| x, y, z | Coordinates of Cartesian coordinate system |
| Y | Deflection at any distance x |
| y | Position of a plane from neutral axis |
| Y_0 | Maximum deflection |
| σ | Stress |
| ε | Axial strain |
| ε_0 | Strain rate corresponding to $\sigma_{dy} = 2\sigma_y$ |
| η | Non-dimensional constant |
| $\left(= \frac{2\dot{\varepsilon}_{or}}{d} \right)$ | |
| θ | Support rotation angle |
| ρ | Density |
| σ_y | Static yield stress |
| σ_{dy} | Dynamic yield stress |
| v | Non-dimensional transverse velocity |
| $\left(= \frac{V}{L/T_0} \right)$ | |

$$v_0 \left(= \frac{V_0}{L/T_0} \right)$$

Non-dimensional transverse initial velocity of colliding mass



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