Optimal PID controller design for level control of three tank system

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Abstract

In this paper, an optimal proportional–integral–derivative controller (PID) controller is designed for level control of three tank system. The integral-square-error (ISE) based approach is used for tuning of PID controller. The ISE of unit step response is minimized for obtaining optimal controller settings. The ISE is constructed in terms of alpha and beta parameters. The teacher learner based optimization (TLBO) technique is used for minimizing the ISE. The TLBO is recently proposed optimization algorithm which is free from algorithm-specific parameters. The results obtained using proposed technique is compared with other well-known methods. It is found that the proposed technique performs much better when compared to others.

Keywords

ISE, Optimization, PID controller, Teacher learner based optimization, Tuning.

1. Introduction

The PID controller is still very much popular in industry because of its simplicity and reliability. Many methods have been proposed in the literature for tuning of parameters of PID controller. One of these methods is Ziegler-Nichols (ZN) settings which is a rule based design criterion [1]. Some other rule based methods are integral of square time weighted error (ISTE), Pessen integral of absolute error (PIAE), Kessler Landau Voda (KLV), some overshoot rule (SO-OV), no overshoot rule (NO-OV), Mantz-Tacconi Ziegler-Nichols (MT-ZN) and refined Ziegler-Nichols (R-ZN) [2].The rule based tuning methods generally provide satisfactory response. The demand of improved dynamic response evolved the various evolutionary computation based tuning techniques. Some of these include Luus-Jaakola optimization procedure for PID controller tuning [3], PID controller tuning based on particle swarm optimization [4], evolutionary computation based PID tuning [5], PID controller tuning based on genetic algorithm [6], PID tuning using soft computing techniques [7], etc. This paper proposes an evolutionary computation based tuning method. The teacher learner based optimization (TLBO) technique is used for obtaining the controller parameters. Being simple and efficient, TLBO is applied for obtaining optimal results for various engineering problems [8-11].

An additional advantage of this algorithm is that it is free from algorithm-specific parameters. The design criterion is ISE of unit step input. The ISE is derived from alpha and beta tables. This method requires only a fixed number of alpha and beta parameters unlike other techniques where the integration up to infinite time is mandatory.

The layout of this brief is as follows. Section 2 describes the three tank system and its model. Section 3 provides the structure of PID controller. The proposed method of tuning is described in section 4. The algorithm utilized to minimize the performance index is discussed in section 5. Section 6 discusses the simulation parameters and quantitative as well as qualitative results obtained. The brief is concluded in section 7.

2. The three tank system

The block diagram shown in Figure 1 illustrates the closed loop control of three tank system [12].

\[ \text{Figure 1: Three tank system} \]
The transfer function of plant i.e. three tank system is given as
\[
G(s) = \frac{K}{(\tau_a s + 1)(\tau_b s + 1)(\tau_c s + 1)}
\]
(1)
Where, \( K \) is gain and \( \tau_a, \tau_b \) and \( \tau_c \) are time constants of three tanks. The \( C(s) \) in block diagram denotes the controller.

3. PID controller
The transfer function of PID controller considered in this work is described as
\[
C(s) = K_p \left( 1 + \frac{1}{T_i s} + T_d s \right)
\]
(2)
Where, \( K_p, T_i \) and \( T_d \) are controller parameters.

The controller settings [2] due to Ziegler-Nichols (ZN), Pessen integral of absolute error (PIAE), some overshoot rule (SO-OV), no overshoot rule (NO-OV) and Mantz-Taconi Ziegler-Nichols (MT-ZN) criteria are provided in Table 1. The parameter \( K_u \) denotes the ultimate gain and the parameter \( T_u \) represents the period for which ultimate gain occurs.

<table>
<thead>
<tr>
<th>S.N.</th>
<th>Rule</th>
<th>( K_u )</th>
<th>( T_u )</th>
<th>( T_d )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>ZN</td>
<td>0.6</td>
<td>0.5T_u</td>
<td>0.125T_u</td>
</tr>
<tr>
<td>2</td>
<td>PIAE</td>
<td>0.7</td>
<td>0.4T_u</td>
<td>0.15T_u</td>
</tr>
<tr>
<td>3</td>
<td>SO-OV</td>
<td>0.33</td>
<td>0.5T_u</td>
<td>0.33T_u</td>
</tr>
<tr>
<td>4</td>
<td>NO-OV</td>
<td>0.2</td>
<td>0.5T_u</td>
<td>0.33T_u</td>
</tr>
</tbody>
</table>

The proposed method
In proposed method, ISE of unit step response is minimized. The error obtained in s-domain for system given in Figure 1 is
\[
E(s) = R(s) - Y(s) = \frac{R(s)}{1 + G(s)C(s)}
\]
where,
\[
R(s) = \frac{1}{s}
\]
(4)
The transfer function of error can be written as
\[
E(s) = \frac{\sum b_i s^{n-i} + \cdots + b_n}{a_0 s^n + a_{n-1} s^{n-1} + \cdots + a_n}
\]
(5)
The ISE is given as
\[
J = \int_{t=0}^{\infty} e^2(t) dt
\]
which in terms of alpha and beta parameters [13] becomes
\[
J = \frac{1}{2} \sum_{i=1}^{n} \beta_i^2
\]
(6)
where \( n \) is the order of error in s-domain (i.e. \( E(s) \) as given by (5))

The alpha and beta parameters are obtained from alpha table and beta table as given in Table 2 and Table 3.
5. Teacher learner based optimization (TLBO) algorithm

The TLBO algorithm [14] consists two phases: teacher phase and learner phase. In teacher phase, the teacher tries to improve the students’ performance to his level. In learner phase, a student interacts to other student of the class to improve his knowledge.

Suppose, there is a total $R$ students in the class and the subjects offered to the students are $C$. The performance of $i$th student, $i=1,2,\ldots, R$, in $j$th subject, $j=1,2,\ldots, C$ is $X_{i,j}$. The teacher tries to improve the performance of students as given below:

$$newX_{i,j} = X_{i,j} + r(X_{best,j} - T_f \bar{X}_{i,j})$$

where $newX_{i,j}$ is updated performance of the class and $r$ is a random number in the range $(0, 1)$. $X_{best,j}$ and $\bar{X}_{i,j}$ denote respectively the performance of best student (i.e. teacher) in the class and mean performance of the class. The parameter $T_f$ is a random number in the range $(1, 2)$.

The teacher factor is determined heuristically and possesses value either 1 or 2 with equal probability. At the end of teacher phase, $newX_{i,j}$ is updated in $X$ if it is a better solution otherwise old performance i.e. $X_{i,j}$ is retained.

In learner phase, each student interact to other to improve his knowledge as given by

$$newX_{i,j} = X_{i,j} + r(X_{p,j} - X_{q,j})$$

where $p$ and $q$ are two randomly selected students of the class such that $p \neq q$, $f(X_{p,j}) < f(X_{q,j})$ and $r = (0, 1)$.

At the end of learner phase, the solution $newX_{i,j}$ is updated in $X$ if it is better otherwise $X_{i,j}$ is retained.

6. Results and discussion

The parameters [12] of three tank system considered in this work are $K = 6$, $\tau_a = 2$, $\tau_b = 4$ and $\tau_c = 6$. For these values, (5) becomes

$$E(s) = \frac{b_3 s^3 + b_2 s^2 + b_1 s + b_0}{a_3 s^3 + a_2 s^2 + a_1 s + a_0}$$

Where,

$$a_0 = T_i \tau_a \tau_b \tau_c \quad a_3 = T_i (1 + K K_p) \quad a_1 = \frac{ijX_{best,j} + r(\bar{X}_{i,j})}{T_f}$$

Table 2 and Table 3 turn out to be, respectively, Table 4 and Table 5.

Table 4 Alpha table

<table>
<thead>
<tr>
<th>$c_0$</th>
<th>$c_1$</th>
<th>$c_2$</th>
<th>$c_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_0$</td>
<td>$a_1$</td>
<td>$a_2$</td>
<td>$a_3$</td>
</tr>
<tr>
<td>$c_4$</td>
<td>$0$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$\alpha_i = c_0 / c_1$

$\alpha_0 = c_0 / c_1$

$\alpha_2 = c_2 / c_3$

$\alpha_3 = c_2 / c_3$

$\alpha_4 = c_2 / c_3$

Table 5 Beta table

<table>
<thead>
<tr>
<th>$d_0$</th>
<th>$d_1$</th>
<th>$d_2$</th>
<th>$d_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b_1$</td>
<td>$b_2$</td>
<td>$b_3$</td>
<td>$b_4$</td>
</tr>
</tbody>
</table>

$\beta_i = d_0 / c_1$

$\beta_0 = d_0 / c_1$

$\beta_2 = d_0 / c_1$

$\beta_3 = d_0 / c_1$

$\beta_4 = d_0 / c_1$

The performance index, given by (7) becomes

$$J = \frac{1}{2} \sum_{i=1}^{4} \frac{\beta_i^2}{\alpha_i}$$

$$J = \frac{1}{2} \sum_{i=1}^{4} \frac{(b_i / a_i)^2 + (b_i^2 / (a_2 - \alpha_i a_2))^2}{a_0 / a_1 + (a_2 - \alpha_i a_2)}$$

For these values, (5) becomes

$$\frac{(b_3 - \beta_a a_3) / (a_3 - \alpha_3 a_3)}{(a_3 - \alpha_3 a_3) / a_3}$$

$$\left( \frac{(b_3 - \beta_a a_3) / (a_3 - \alpha_3 a_3)}{(a_3 - \alpha_3 a_3) / a_3} \right)$$

...
Minimizing (22) using TLBO, the optimal results obtained are given in Table 6. Table 6 also provides the settings obtained due to ZN, PIAE, SO-OV, NO-OV and MT-ZN rules.

### Table 6 Controller parameters

<table>
<thead>
<tr>
<th>S.N.</th>
<th>Rule</th>
<th>$K_p$</th>
<th>$T_i$</th>
<th>$T_d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Proposed</td>
<td>39.62</td>
<td>948.13</td>
<td>0.23</td>
</tr>
<tr>
<td>2</td>
<td>ZN</td>
<td>1</td>
<td>6.28</td>
<td>1.57</td>
</tr>
<tr>
<td>3</td>
<td>PIAE</td>
<td>1.17</td>
<td>5.03</td>
<td>1.89</td>
</tr>
<tr>
<td>4</td>
<td>SO-OV</td>
<td>0.55</td>
<td>6.28</td>
<td>4.15</td>
</tr>
<tr>
<td>5</td>
<td>NO-OV</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>MT-ZN</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure 2 provides the step responses obtained due to proposed method and other settings. The settling time and peak over shoot are tabulated in Table 7.

From the results obtained, it is clear that the proposed method gives much better controller setting when compared to other methods.

### Table 7 Settling time and peak overshoot

<table>
<thead>
<tr>
<th>S.N.</th>
<th>Rule</th>
<th>$T_s$ (Sec.)</th>
<th>$M_p$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Proposed</td>
<td>0.003</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>ZN</td>
<td>11.90</td>
<td>8.96</td>
</tr>
<tr>
<td>3</td>
<td>PIAE</td>
<td>11.50</td>
<td>6.97</td>
</tr>
<tr>
<td>4</td>
<td>SO-OV</td>
<td>13.90</td>
<td>5.00</td>
</tr>
<tr>
<td>5</td>
<td>NO-OV</td>
<td>15.60</td>
<td>7.29</td>
</tr>
<tr>
<td>6</td>
<td>MT-ZN</td>
<td>11.90</td>
<td>8.96</td>
</tr>
</tbody>
</table>

7. Conclusion

In this work, an optimal PID controller is proposed for level control of three tank system. The controller tuning is achieved by minimizing the ISE of unit step response. The ISE is constructed with the help of alpha and beta parameters. The teacher learner based optimization algorithm is utilized for minimizing the ISE. The obtained controller settings show better performance in terms of qualitative and quantitative results.

### Acknowledgment

None.

### Conflicts of interest

The authors have no conflicts of interest to declare.

### References


