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Hybrid beam-forming techniques for multi-cell massive MIMO

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Abstract
Various hybrid beam-forming research works are currently underway, as the cost and power consumption of fully digital beam-forming limit its applicability in the fifth generation (5G) massive multiple input multiple output (MIMO). A majority of studies assumed that the channel state information (CSI) is perfect, the hardware is ideal and the network is represented as a single-cell scenario. As a result, studies incorporating Kalman-based precoders with existing beam-formers for multi-cell systems under non-ideal conditions warrant further investigation. This study aimed to achieve the performance analysis of Kalman hybrid beam-forming technique for multi-cell massive MIMO under hardware impairment and imperfect CSI. The spectral efficiency (SE) of the multi-cell based Kalman hybrid beam-forming technique has been analysed along with zero forcing, minimum mean square error (MMSE), and mean square error (MSE) precoders. It can be seen from the MATLAB simulation that the SE of fully digital MSE, Kalman, MMSE, and zero forcing precoders at signal to noise ratio (SNR) of 20 dB and number of transmitting antennas of 128 under non-ideal conditions is 8 bps/Hz, 7 bps/Hz, 5.2 bps/Hz, and 6 bps/Hz respectively. The SE of fully digital, Kalman, MMSE, and zero forcing precoders at SNR of 20 dB for 128 transmitting antennas under ideal conditions is 11 bps/Hz, 9.6 bps/Hz, 7.2 bps/Hz, 8.1 bps/Hz respectively. Additionally, the simulation shows that when the number of transmitting antennas rises and the number of users falls, the SE of beam-formers increases. Increasing the number of transmitting antennas and decreasing the number of users mitigates the effect of hardware impairments and imperfect CSI in a multi-cell based linear beam-formers. Imperfect hardware and CSI, in general, degrade the SE of multi-cell multi-user based linear beam-formers.

Keywords
Hybrid beam-forming, Kalman precoder, Massive MIMO, Minimum mean square error precoder, Millimeter wave, Non-ideal conditions, Zero forcing precoder.

1.Introduction
The significantly increasing mobile capacity demand due to the introduction of various innovative data services and highly capable and affordable mobile devices has brought research communities to investigate millimeter wave, massive multiple input multiple output (MIMO) and beam-forming [1−4]. Millimeter waves have been introduced to overcome channel spacing scarcity, which can be seen at lower frequency ranges [3]. Massive MIMO is a technology in which a high number of antennas are installed at the transmitter to boost capacity by transmitting multiple data streams [3]. Beam-forming antenna array technologies are being integrated with millimeter wave massive MIMO to provide high capacity and better signal to noise ratio (SNR).

Commonly known beam-forming techniques are analog, digital, and hybrid. Analog beam-forming, which supports single data streams, exhibits low performance and is not feasible to be used in the fifth generation (5G).

Digital beam-forming, which requires a separate radio frequency (RF) chain for every antenna, introduces high cost and power consumption. Due to the high cost and power consumption of fully digital beam-forming and the poor performance of analog beam-forming, hybrid beam-forming has drawn the attention of actors in the communication industry [4]. Generally, massive MIMO performance depends on hardware perfection and channel state information (CSI) availability, which are critical components to influencing the performance. The majority of hybrid beam-forming research has concentrated on perfect CSI and ideal hardware-based precoders/combiners

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[5−8]. Moreover, research work is mostly considered a single-cell single-user or multi-user system [9–12]. However, in a practical system, complete CSI and ideal hardware are challenging to obtain. Some hybrid beam-forming research has been conducted under non-ideal conditions, such as imperfect CSI [13−17], hardware impairments [18−20] and joint imperfect CSI and hardware [21−23]. Work involving a Kalman-based precoder for multi-cell systems under non-ideal conditions, on the other hand, requires further investigation. The mathematical formulation and performance analysis of hybrid precoding/combining for multi-cell systems under imperfect CSI has been developed in this research, considering the hardware impairment effect, channel estimation errors, multi-user interference, and inter-cell interference.

The major contributions of this work are summarized as follows:

- For performance analysis, a digital combiner is added to the existing Kalman-based beam-former, which was previously solely considered an analog combiner.
- A single-cell multi-user Kalman-based beam-former is extended to a multi-cell multi-user scenario and its performance is evaluated.
- The effect of hardware impairments on the performance of hybrid beam-forming for multi-cell systems under imperfect CSI was investigated.

The rest of the paper is laid out as follows. Section-2 focuses on a literature review. Section-3 presents the system and channel model. Section-4 presents results of multi-cell-based linear beam-formers under non-ideal conditions. Sections -5 and -6 offer the paper's discussion and conclusion, respectively.

2.Literature review

This section provides a detailed review of beam-former design and performance analysis. In a multi-user scenario, the impact of joint hardware impairments and CSI, which is imperfect on the Kalman beam-former, has been studied in [1]. The study found that impairments considerably decrease the affordable complexity beam-former's spectral efficiency (SE). However, in this study, the multi-cell environment was not considered. Taking into account the residual additive transceiver hardware impairments (RATHIs) and the amplified thermal noise (ATN), a single-cell-based hybrid precoder and combiner that includes both digital and analog processing has been devised [24]. The study has demonstrated how much the hardware impairment reduces the practical system's SE. However, when it demonstrated SE degradation under imperfections, it did not include the multi-cell environment or CSI imperfections. The performance of single-cell-based systems in realistic scenarios has been addressed in [25]. When imperfect channel state information (I-CSI), successive interference cancellation (SIC), which is imperfect successive interference cancellation (I-SIC), and non-ideal hardware were all present, it looked at how a full-duplex (FD) MIMO relay system worked. The study mathematically obtained the accurate closed-form equations of the ergodic capacity. However, with imperfect CSI and hardware limitations, multi-cell based precoding/combining has not been studied. A single base station (BS) with multi-users and a general downlink model with zero-forcing precoding that could be used in realistic heterogeneous networks having multiple antennas at the BSs has been considered in [18]. The study invoked imperfect CSIT as a result of pilot contamination, channel aging as a result of users' relative movement, and RATHIs, which is unavoidable. For performance analysis, however, multi-cell and other linear precoding approaches were not considered. The hardware impairments' impact on the downlink massive MIMO performance has been explored in [26]. The study used maximum ratio transmission (MRT) precoder under a single-cell system and all of the hardware impairment characteristics, but it didn't take into account the multi-cell environment or imperfect CSI, as we did in this study. The RATHs' impact on the MIMO relay systems' ergodic capacity was explored in [27]. The ergodic channel capacity of relay systems has been thoroughly characterized. However, imperfect CSI has not been considered. A single-cell realistic multiple input single output (MISO) broadcast channel, which is restricted by hardware limitations, has been addressed in [28]. The study also experimentally confirmed a model of hardware impairments that included multiplicative distortion. At the transmitter, it looked at the potential robustness of rate splitting (RS) to each individual hardware flaw in ideal and non-ideal channel conditions. However, in this study, multi-cell environments and massive MIMO were not considered. Having a non-ideal channel at the BS, the study in [6] addressed robust symbol level precoding (SLP) design for the downlink MISO channels. Hardware limitations, multi-cell, and massive MIMO, on the other hand, were not considered. The coarsely quantized multi-user multiple input multiple output (MU-MIMO) system precoding problem was investigated in [29]. A random matrix having secondorder statistics, which is finite, is used to describe the channel uncertainties. Hardware impairments, on the other hand, were not considered in this study. The channel aging impact on the single-cell-based uplink and downlink performance of frequency division duplex (FDD) massive MIMO as the dimension of the system rises was investigated in [30]. However, hardware impairment and a multi-cell environment were not considered. The performance of hybrid Kalman-based beam-forming for millimeter wave massive MIMO using multi-cell and multi-users was compared to that of conventional linear beam-formers in [31]. Each linear beam-former has a mathematical model for SE under a multi-cell, multi-user scenario, and the performance has been demonstrated using MATLAB software. However, imperfect CSI and hardware impairments have not been taken into account. The performance of hybrid beam-forming for multi-cell multi-user millimeter wave massive MIMO has been investigated in [32]. Analog beam-forming is acquired first to optimize the beam-forming gain, and then digital beam-forming is calculated using the weighted minimum-mean-square-error (wMMSE) approach. Simulation results show that the suggested algorithms are successful and can outperform other state-of-art algorithms. However, the Kalman algorithm and non-ideal conditions have not been considered. In [33], a three-stage hierarchical technique that evolved a hybrid precoder/combiner design under imperfect CSI and a single-cell scenario has been developed. The sum-rate results show the performance enhancements brought about by their approach in comparison to previous hybrid precoders/combiners under the same scenario. However, hardware impairment, Kalman beam-former, and multi-cell system have not been considered. In [34], a novel channel estimate and hybrid beam-forming design method while considering switches and non-ideal phase shifters have been considered. However, the multi-cell cell scenario has not been discussed in this research. In [35‒38], single-cell-based hybrid beam-forming under perfect CSI and hardware impairment has been considered. Non-ideal conditions, multi-cell scenarios, and Kalman-based beam-former have not been considered. In [39], the combined effects of channel non-reciprocity, quantized phase shifters (QPSs), and channel estimation errors on the hybrid beam-forming system's possible SE have been examined. However, the multi-cell scenario has not been considered. In [40], the performance of hybrid precoding techniques, including Kalman-based precoder for single-cell multi-user environments, has been investigated. However, impairment incorporated multi-cell multi-user scenario has not been considered.

The research works on the aforementioned literature and investigates hybrid precoders' and combiners' performance for massive MIMO. Studies looked at a variety of topics, including hardware impairments, single-cell situations, and imperfect CSI. There is no literature that contains a Kalman-based precoder for multi-cell environments with imperfect CSI and hardware.

In this section, beam-formers that exist in the literature have been discussed. We found that precoding schemes for millimetre wave massive MIMO under non-ideal conditions and single-cell scenarios, including Kalman-based beam-formers, have been addressed but rarely considered in multi-cell environments. We, hence, proposed Kalman-based precoding under a non-ideal multi-cell multi-user environment to analyse its performance. In the next section, we developed a system model in order to formulate the problem, define the solution criteria and apply deep insights towards the solution.

3.Methods

3.1System and channel model

In this subsection, mathematical formulations of SE of extended work of [1, 31] for different precoding techniques have been presented. This work constitutes imperfect CSI and imperfect hardware such as RATHI deficiencies and ATN for different precoding techniques for multi-cell multi-user environments. To achieve aforementioned goal, system model and channel model has been explained first and precoding techniques SE mathematical formulations for multi-cell multi-user environment under non-ideal conditions has followed.

System model

The model in *Figure 1(a and b)* has been used in this research. It extended the work on [1] from a single-cell multi-user scenario to a multi-user scenario that incorporates imperfect CSI and hardware impairments. With this model, the BS transmits Ns data streams through N_{RF} RF chains and N_T antennas for serving K users with a given N_R antenna and partial RF chain ($N_S < N_{RF} < N_T$).

The terms for transmission point (TP) i and user k in cell l can be defined in the *Table 1*:

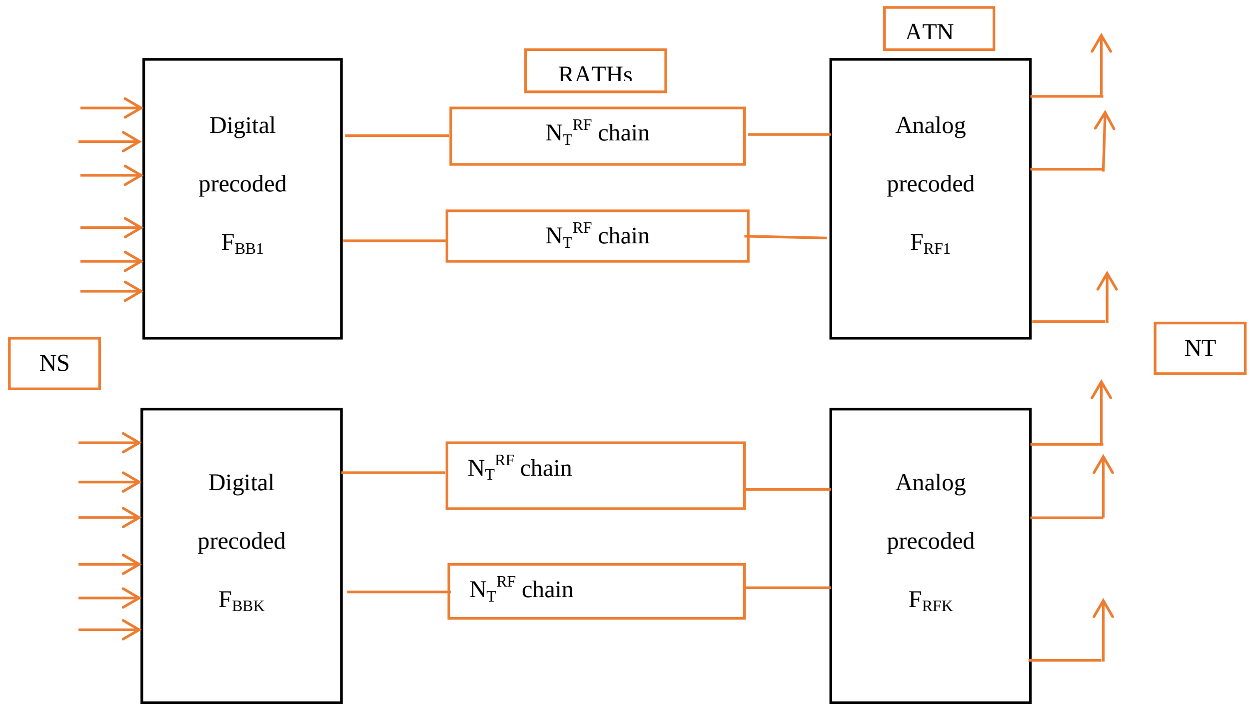

Figure 1(a) Hybrid precoding structure for multi-cell multi-user millimeter wave massive MIMO system under non-ideal conditions

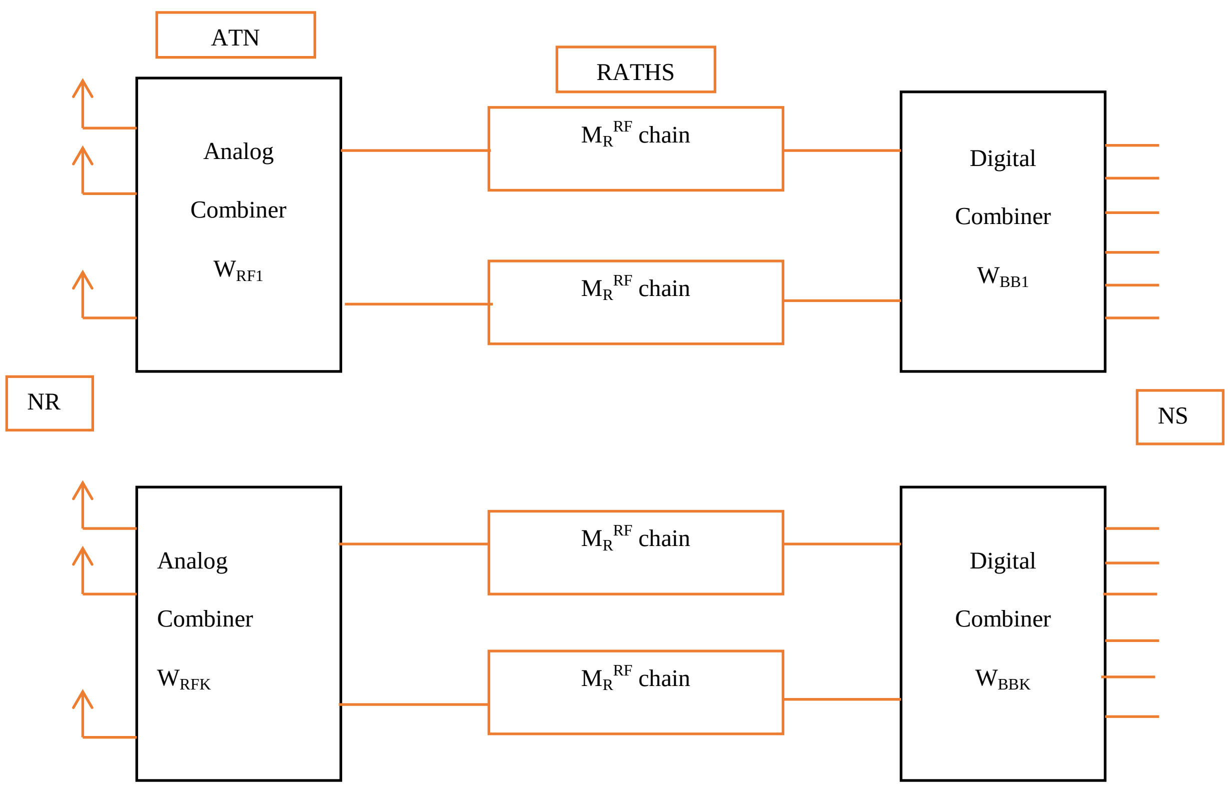

Figure 1(b) Hybrid combiner for multi-cell multi-user massive MIMO under non-ideal conditions

Table 1 The precoding and combining matrix notations and their equivalence

Notations	Equivalent	Description
$N_T \times N_T^{RF}$	$F_{RFk,l}$	Matrix for RF precoding
$N_T^{RF} \times N_S$	$F_{BBk,l}$	Matrix for baseband precoding
$M_T^{RF} \times N_S$	$W_{BBk,l}$	Matrix for base band combining
$N_R \times N_R^{RF}$	$W_{RFk,l}$	Matrix for RF combining
$N_R \times N_T$	$\hat{H}_{i,k,l}$	Imperfect CSI incorporated downlink channel

where:

N_T^{RF} - Total number of RF chains at each TP

M_T^{RF} - The number of RF chain connected to base band precoder for one user.

N_R - Receiving antennas with N_R^{RF} chains

N_R^{RF} - RF chains at each digital baseband user

N_T - The number of transmitting antennas

Ns - Number of data stream per user

Based on the work in [1], by considering the downlink transmission and assuming that a single carrier waveform is used, the discrete-time transmitted signal at sample time-interval n is given by Equation 1:

$$x_k = F_{RF} F_{BB} s_k \tag{1}$$

The received signal at the mobile station (MS-m) is given by Equation 2:

$$r_k = (W_{RF} W_{BB}) H_k F_{RF} F_{BB} S_K + n W_{RF} W_{BB} \tag{2}$$

Channel model

From the work in [2], the imperfect channel is modelled as (Equation 3 and 4):

$$H_k = \hat{H}_k - \Delta_k \tag{3}$$

$$\hat{H}_k = H_k + \Delta_k \tag{4}$$

The imperfect and ideal channels are represented by Ĥ and H, respectively. The CSI stochastic error, which is considered to be independent of the perfect channel, is represented by Δ. The imperfect channel is the summation of the perfect channel and error. The perfect channel can be obtained by subtracting the error from the imperfect channel.

The received signal at cell L and user K with transceiver hardware impairment and imperfect CSI for a multi-cell multi-user scenario is given by Equation 5:

$$r_{mkl} = (H_k + \Delta_k)(X_k + \xi_k + \varepsilon_k) + \sum_{i\neq kl}^{k} (H_k + \Delta_k)(X_n + \xi_n + \varepsilon_n) + z_n \tag{5}$$

Where

ξ_m = RATHI at the transmitter

ξ_n = RATHI at the receiver

ε_m = ATN at the receiver

ε_n = ATN at the transmitter

After applying the baseband combiner $W_{RF} W_{BB}$, the expanded solution of Equation 5 becomes Equation 6:

$$r_{mkl} = W_{RF} + W_{BB}((H_k + \Delta_k)(F_{RF} F_{BB} S_k(\xi_k + \varepsilon_k)) + \sum_{i\neq kl}^{k} (H_k + \Delta_k)(F_{RF} F_{BB} S_n(\xi_n + \varepsilon_n) + z_n) \tag{6}$$

Distributing Equation 6 to show the desired and interference plus noise signal yields Equation 7:

$$r_{mkl} = W_{RF} W_{BB}(H_k + \Delta_k)(F_{RF} F_{BB} S_k + W_{RF} W_{BB}(H_k + \Delta_k)(\xi_k + \varepsilon_k) + W_{RF} W_{BB} \sum_{i\neq kl}^{k} (H_k + \Delta_k)(F_{RF} F_{BB} S_n + (\xi_n + \varepsilon_n) + W_{RF} W_{BB} z_n \tag{7}$$

The channel matrix for multi-cell multi-user based linear precoders under non-ideal conditions can be shown in Equation 8:

$$\hat{H}_k = \left[\sqrt{\frac{N_{bskl} N_{mskl}}{N_{el} N_P}} \sum_{l=1}^{N_C l} \sum_{k=1}^{nl} \alpha^{\ell K}{}_{\ell K} \partial_R (\phi_{K_R}^{\ell_K}, \theta_{K_R}^{\ell_K}) \partial_T^H (\phi_{\ell_T}^{\ell_K}, \theta_{\ell_T}^{\ell_K}) \right] + \Delta_k \tag{8}$$

The gain (complex) of the path k in the cluster l is denoted by $\alpha_{l,k}^{(l,k)}$ while N_{cl} and Np, which stand for the number of scattering clusters and paths per cluster, respectively. $\phi_{k,R}^{(l,k)}$ and $\Theta_{k,R}^{(l,k)}$ represents angles of arrival (AoAs) of azimuth and elevation at receiver, while $\phi_{k,T}^{(l,k)}$ and $\phi_{k,T}^{(l,k)}$ are the corresponding angles of departure (AoDs) at transmitter. The transmit and receive array response vectors of the antenna are denoted by ðT and ðR, respectively. It is expected that the BS and each MS are familiar with the antenna arrays' geometry. The results presented in this manuscripts can be applied to uniform phased arrays (UPA) whose array response vectors expressed in terms of wavelength of the signal and inter element distance.

In this subsection, we have developed a system model for the proposed precoding solutions under impairment incorporated multi-cell multi-user environment. In the next subsection, we will present SE expressions for different beam-forming algorithms.

3.2Multi-cell based linear beam-formers under non-ideal conditions

This article develops a mathematical model that shows the effect of joint hardware impairments and imperfect CSI on multi-cell multi-user based hybrid precoders.

Kalman-based hybrid precoder

The error e(n) at the n[th] Kalman iteration for multi-cell systems can be calculated using the work on [1] as follows (Equation 9):

$$e(n)=\frac{s(n)_{k,l}-\overline{s}(n)_{k,l}}{\left\|s(n)_{k,l}-\overline{s}(n)_{k,l}\right\|_F^2} \quad (9)$$

Incorporating the baseband combining matrix $W_{BB,}$ the state equation for Kalman-based hybrid multi-cell multi-user system can be shown in Equation 10.

$$W_{BBk,l}(n/n)=W_{BBk,l}(n/n-1)+K(n)_{k,l}E\{diag[e(n)]\} \quad (10)$$

The diagonal matrix error representation of multi-cell multi-user expressed in Equation 10 is given by Equation 11:

$$E\{diag[e(n)]\}=\frac{I-\widehat{H}_{ek,l}W_{BBk,l}(n/n-1)}{\left\|I-\widehat{H}_{ek,l}W_{BBk,l}(n/n-1)\right\|_F^2} \quad (11)$$

Substituting Equation 11 to Equation 10 yields Equation 12:

$$W_{BBk,l}(n/n)=W_{BBk,l}(n/n-1)+K(n)_{k,l}\frac{I-\hat{H}_{ek,l}W_{BB}(n/n-1)}{\left\|I-\hat{H}_{ek,l}W_{BB}(n/n-1)\right\|_F^2} \quad (12)$$

The hybrid millimeter wave combining matrix can be formulated using the Kalman hybrid multi-cell-based approach that minimizes error e(n) as follows (Equation 13 and 14):

$$\begin{aligned}&\min imize\ \mathrm{E}\{\|S_{k,l}-W_{BBk,l}^{H}W_{RFk,l}^{H}\overline{S}_{k,l}\|^2\}\\&subject\ \text{to}\ \mathrm{W}_{RFk,l}\in W_{RFk,l}and\mathrm{W}_{BBk,l}\end{aligned} \quad (13)$$

The effective channel can be:

$$h_{k,l}^{H}=(W_{BBk,l}^{H}W_{RFk,l}^{H})\hat{H}_{ek,l}F_{RFk,l} \quad (14)$$

The achievable sum rate of Kalman-based solution under imperfect CSI and hardware impairment in a multi-cell multi-user scenario can be shown in Equation 15.:

$$SE_{Kl}=\sum_{n=1}^{k}\log_2\left(1+\frac{P_{Kl}\left|W_{BB}^{H}W_{BB}^{H}(H_k+\Delta_k)F_{RF}F_{BB}+(\xi_k+\varepsilon_k)\right|}{m_{Kl}\frac{P_{Kl}}{M_{Kl}}\sum_{l\neq K}^{K}\left|W_{BB}^{H}W_{BB}^{H}(H_k+\Delta_k)F_{RF}F_{BB}+(\xi_n+\varepsilon_n)\right|^2+\delta^2+\sum_{il\neq kl}^{h}\beta i_l}\right) \quad (15)$$

Zero forcing hybrid precoder

The achievable sum rate for multi-cell multi-user based zero forcing hybrid precoding under non-ideal conditions by taking basic ideas from [41, 42] can be shown in Equation 16 and 17.:

$$(S_{\in ZF})_{Kl}=Log_2(1+SINR) \quad (16)$$

Where,

$$SINR=\frac{\beta}{\left\|F^{2F}\right\|^2},\left\|F^{2F}\right\|^2=(F_{RF}W^H)(F_{RF}W) \quad (17)$$

Where,

$$(F_{RF}W)^H = (F_{RF}(H_k+\Delta_k)^H((H_k+\Delta_k)F_{RF}\overset{H}{F_{RF}}(H_k+\Delta_k))^{-1})^H \overset{H}{F_{RF}} \quad (18\text{-a})$$

$$(F_{RF}W)^H = ((H_k+\Delta_k)F_{RF})^{-1})^H \overset{H}{F_{RF}} = (H_k+\Delta_k)^{-1})^H \quad (18\text{-b})$$

$$F_{RF}W = F_{RF}\overset{H}{F_{RF}}(H_k+\Delta_k)((H_k+\Delta_k)F_{RF}\overset{H}{F_{RF}}(H_k+\Delta_k)^H)^{-1} \quad (19\text{-a})$$

$$F_{RF}W = F_{RF}((H_k+\Delta_k)F_{RF})^{-1}F_{RF} = (H_k+\Delta_k)^{-1} \quad (19\text{-b})$$

Substituting Equation 18 and Equation 19 into Equation 17 yields:

$$SINR = \frac{\beta}{((H_k+\Delta_k)^{-1})^H(H_k+\Delta_k)^{-1}} = \frac{\beta}{((H_k+\Delta_k)(H_k+\Delta_k)^H)^{-1}} \quad (20)$$

Mean square error (MSE) fully digital precoder

From the work on [43, 44], the sum-MSE equation can be expressed as Equation 21:

$$E\left\|s-\bar{s}\right\|^2 = \| I-\hat{H}_{FD}F_{RF_{FD}}F_{BB_{FD}}\|_F^2 \quad (21)$$

The hybrid precoder that minimizes this sum-MSE can be obtained by Equation 22:

$$\begin{aligned}&\min imize\ \mathrm{Tr}\{(\mathrm{I}-\hat{H}_{FD}FRF_{FD}FBB_{FD})(I-(Hk_{FD}+\Delta k)FRF_{FD}FBB_{FD})^H\}\\ &subject\ \text{to}\ \|FRF_{FD}FBB_{FD}\|=\mathrm{N_s}\end{aligned} \quad (22)$$

The SE of MSE fully digital precoding is given by Equation 23:

$$SE_{MSE\_fd} = Log\left[\frac{1+\frac{p}{k}\left|W_{RF\_fd}^H F_{RF\_fd}(H_{k\_fd}+\Delta_k)F_{BB\_fd}+(\xi_k+\varepsilon_k)\right|^2}{\sum_{\ell\neq K}^{K}\frac{P}{K}\left|W_{RF\_fd}^H F_{RF\_fd}(H_{k\_fd}+\Delta_k)F_{BB\_fd}+(\xi_n+\varepsilon_n)\right|^2+\delta^2}\right] \quad (23)$$

Hybrid MMSE precoder

For this scheme, an SE formulation has been developed based on different literatures and is used in simulation for performance comparison. Its SE for a multi-cell multi-user scenario under non-ideal conditions can be expressed as Equation 24:

$$SE_{MMSE} = Log_2(1+SINR) \quad (24)$$

$$SNR_{MMSE} = \frac{\frac{P}{K}\left|h_{e_{kl}}F_{BBk}\right|^2}{\sum_{\ell\neq K}^{K}\frac{P}{K}\left|h_{e_{kl}}F_{BBn}\right|^2+\delta} = \frac{\frac{P}{K}\left|W_{RF}^H F_{RF}(H_k+\Delta_k)F_{BB}+(\xi_k+\varepsilon_k)\right|^2}{\sum_{\ell\neq K}^{K}\frac{P}{K}\left|W_{RF}^H F_{RF}(H_k+\Delta_k)F_{BB}+(\xi_n+\varepsilon_n)\right|^2+\delta^2} \quad (25)$$

$$= Log\left[\frac{1+\frac{p}{k}\left|W_{RF}^H F_{RF}(H_k+\Delta_k)F_{BB}+(\xi_k+\varepsilon_k)\right|^2}{\sum_{\ell\neq K}^{K}\frac{P}{K}\left|W_{RF}^H F_{RF}(H_k+\Delta_k)F_{BB}+(\xi_n+\varepsilon_n)\right|^2+\delta^2}\right] \quad (26)$$

Table 2 shows the simulation settings utilized in MATLAB software, which were chosen based on prior publications [8, 13, 42, 45]. We presented the system model and SE's mathematical expression for precoding schemes such as Kalman-based, fully digital MSE, hybrid MMSE, and zero forcing in a multi-cell multi-user environment in this section. In the next section, we will present simulation results for performance comparison.

Table 2 Parameter settings for simulation

Notation	Parameters	Value
N_users	Number of users	8,16
N_cells	Number of cells	4
N_Path	Number of paths	10,16
TX_ant	Number of TX ant	64, 128, 256
RX_ant	Number of RX ant	4, 8, 16
ξ_m	Effect of RATHI TX	0.0156
ξ_n	Effect of RATHI RX	0.0156
ε_m	Effect of ATN TX	0.05
ε_n	Effect of ATN TX	0.05
P_cont	Pilot contamination	0.82
CH_aging	Channel aging	0.05
QN_err	Quantization error	0.25
FD_delay	Feedback delay	0.15
IMP_re	Imperfect channel res	1.5

4.Results

In this section, the performance of impairment-incorporated precoders has been evaluated and compared with that of ideal or perfect conditions. This work mainly focused on showing the performance of zero forcing, MSE fully digital, Kalman precoding, and hybrid MMSE under hardware impairments and imperfect CSI consideration for multi-cell multi-user systems. A hybrid beam-forming for massive MIMO at millimeter wave frequency is investigated using a transmitter with 16 ×16 UPA and four MSs with 4×4 UPA. The simulation platform used was MATLAB R2020a. The MATLAB scripts start by defining system and channel attributes for the proposed system model in order to generate channels for each user. System and channel parameters include the number of antennas in the BS and MS, the number of channel paths, the number of users, SNR values, the wireless channel complex gain, and the AoAs and AoDs beam-steering vectors. AoAs and AoDs are evenly distributed across the range [−π/2, π/2]. The azimuth AoAs/AoDs should be evenly distributed in the range [0, 2π], the elevation AoAs/AoDs should be evenly distributed in the range [−π/2, π/2], and performance analysis should accommodate for incomplete channel knowledge. Four important points can be raised from computed results. I) As shown in *Figures 2* and *3*, the SE of all beam-formers is affected less by increasing, transmitting antennas in a multi-cell multi-user scenario under non-ideal conditions. II) SE of all beam-formers for multi-cell multi-user scenario under non-ideal conditions decreases by increasing the users' number, as can be seen in *Figures 4* and *5*. III) The SE of all beam-formers in a multi-cell multi-user scenario under non-ideal conditions increases as receiver antennas is increased, as shown in *Figures 6* and *7*. IV) The SE of all beam-formers in a multi-cell multi-user scenario under non-ideal conditions decreases as the number of channel paths increases, as shown in *Figures 8* and *9*. Simulation results in *Figure 3* show that all impairment incorporated beam-formers SE are affected less when transmitting antennas (256) increases as compared to *Figure 2*, where the number of transmitting antennas is 128. Solid lines show beam-formers under ideal conditions, while dashed lines show beam-formers under non-ideal conditions. From *Figure 2*, the SE of fully digital MSE, Kalman, MMSE, and zero forcing precoders at an SNR of 20 dB and number of transmitting antennas of 128 under non-ideal conditions is 8 bps/Hz, 7 bps/Hz, 5.2 bps/Hz, and 6 bps/Hz, respectively. From *Figure 3*, the SE of fully digital, Kalman, MMSE, and zero forcing precoders at an SNR of 20 dB and a number of transmitting antennas of 256 under non-ideal conditions is 10 bps/Hz, 8.8 bps/Hz, 6 bps/Hz, and 7.9 bps/Hz, respectively. From *Figure 2* and *Figure 3*, it can be seen that increasing the antenna number from 128 to 256 improves the SE of all beam formers under non-ideal conditions. Such an improvement came mainly due to an increase in multiplexing gain associated with antennas. There is 2 bps/Hz, 1.8 bps/Hz, 0.8 bps/Hz, and 1.9 bps/Hz SE improvement for fully digital MSE, Kalman, MMSE, and zero forcing precoders at an SNR of 20 dB when 256 antennas are employed at the BS and compared to the SE of beam-formers with 128 antennas at BS.

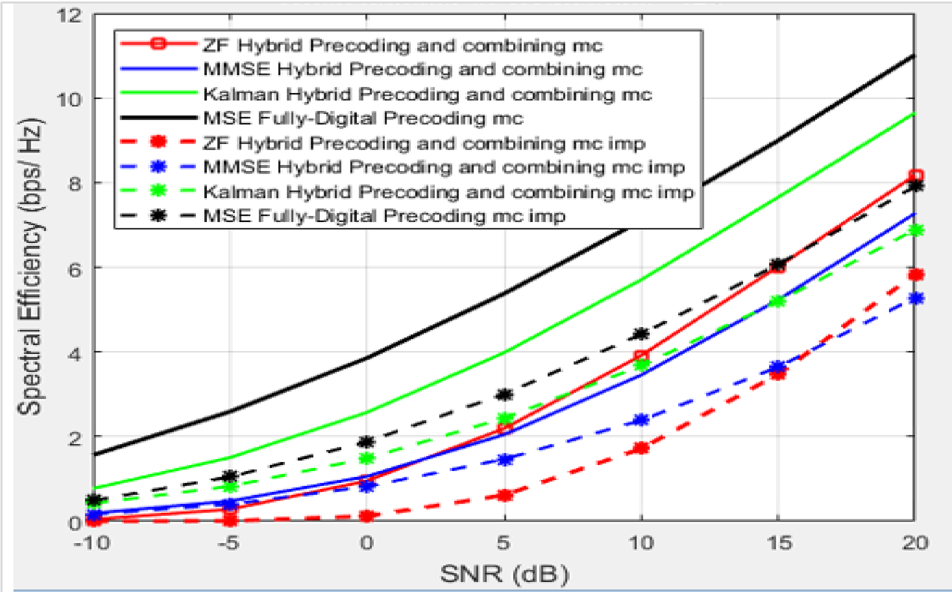

Figure 2 SE for multi-cell-based beam-forming (Nt = 128)

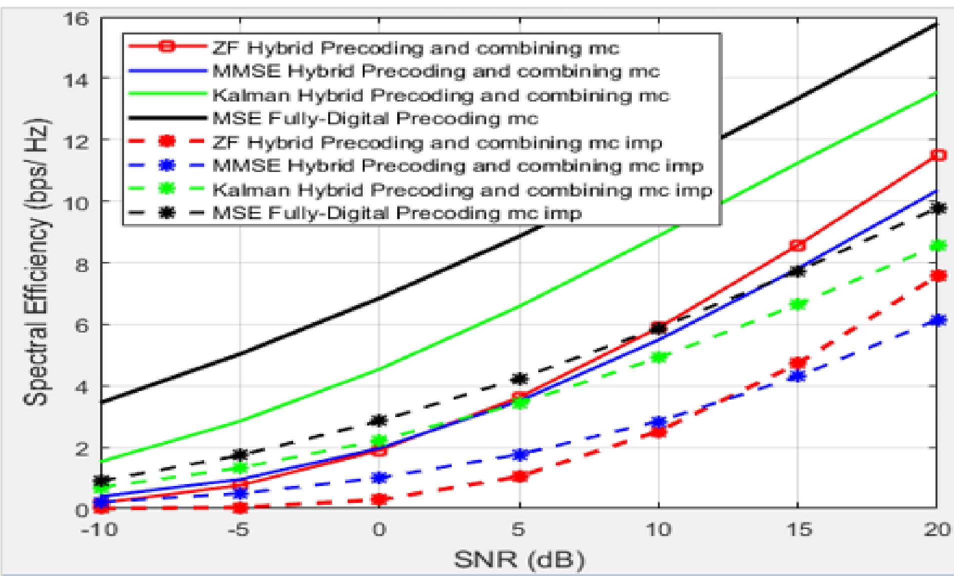

Figure 3 SE for multi-cell-based beam-forming (Nt = 256)

Simulation results in *Figure 4* and *Figure 5* show that all impairment incorporated beam-formers SE experience degradation as the user number under a multi-cell system increases from users = 8 in *Figure 4* to users = 16 in *Figure 5*. From *Figure 4*, the SE of fully digital MSE, Kalman, MMSE, and zero-forcing precoders at SNR of 20 dB and number of users of 8 under non-ideal assumptions is 8 bps/Hz, 7.8 bps/Hz, 5.8 bps/Hz, and 6.4 bps/Hz, respectively. From *Figure 5*, the SE of fully digital, Kalman, MMSE, and zero forcing precoders at an SNR of 20 dB and a number of users of 16 under non-ideal assumptions is 7.6 bps/Hz, 6.9 bps/Hz, 5.3 bps/Hz, and 5.5 bps/Hz, respectively. From *Figure 4* and *Figure 5*, it can be seen that increasing the users' number from 8 to 16 degrades the SE of all beam formers under non-ideal conditions. Such degradation came mainly due to the increased probability of interference associated with increasing users. There is 0.4 bps/Hz, 0.9 bps/Hz, 0.5 bps/Hz, and 0.9 bps/Hz SE degradation for fully digital MSE, Kalman, MMSE, and zero forcing precoders at an SNR of 20 dB when 8 users are served, and compared the SE of beam-formers with 16 users served under a multi-cell system. In this research, it can be considered that the number of data streams is equal to the number of users, and hence the results associated with users similarly show the results of data streams per user.

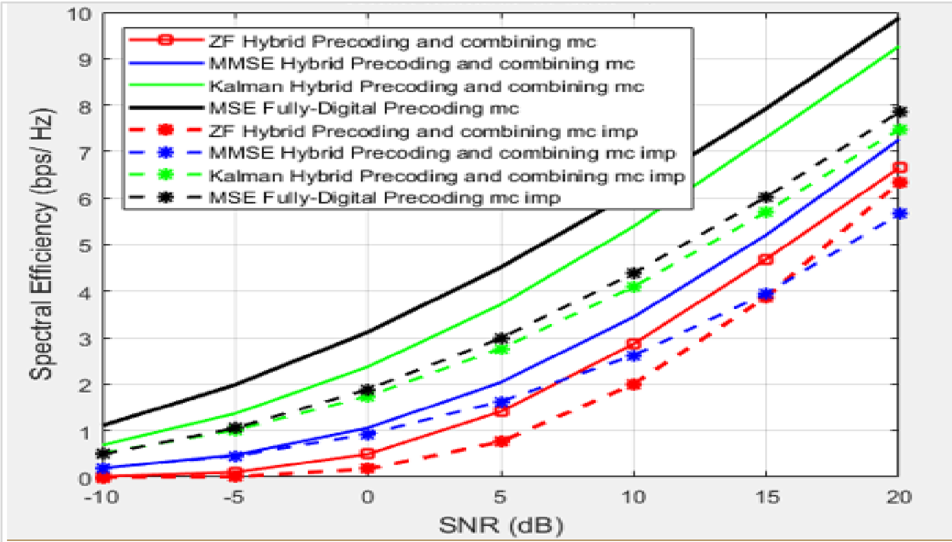

Figure 4 SE for multi-cell based beam-forming (Users = 8)

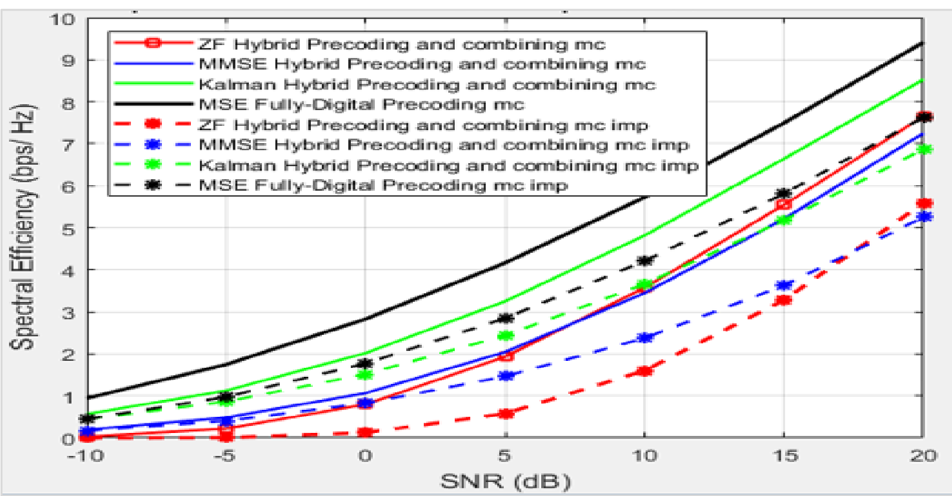

Figure 5 SE for multi-cell based beam-forming (Users = 16)

The beam-formers with multi-cell systems under non-ideal conditions exhibit better SE when the number of receiving antennas increases from 4 antennas per user to 8 antennas per user, as can be seen in *Figure 6* and *Figure 7*. From *Figure 6*, the SE of fully digital, Kalman, MMSE, and zero forcing precoders at an SNR of 20 dB and a number of receive antennas of 4 per user under non-ideal conditions is 7.2 bps/Hz, 6.9 bps/Hz, 5.2 bps/Hz, and 5.1 bps/Hz, respectively. From *Figure 7*, the SE of fully digital, Kalman, MMSE, and zero forcing precoders at an SNR of 20 dB and a number of receive antennas of 8 per user under non-ideal conditions is 7.6 bps/Hz, 7.2 bps/Hz, 5.32 bps/Hz, and 5.3 bps/Hz, respectively. From *Figure 6* and *Figure 7*, it can be seen that increasing the antenna number per user from 4 to 8 improves the SE of all beam formers under non-ideal conditions. Such an improvement came mainly due to an increase in multiplexing gain associated with antennas. There is a 0.4 bps/Hz, 0.3 bps/Hz, 0.12 bps/Hz, and 0.2 bps/Hz SE improvement for fully digital MSE, Kalman, MMSE, and zero forcing precoders at an SNR of 20 dB when 8 antennas per user are employed and compared to the SE of beam-formers with 4 antennas per user.

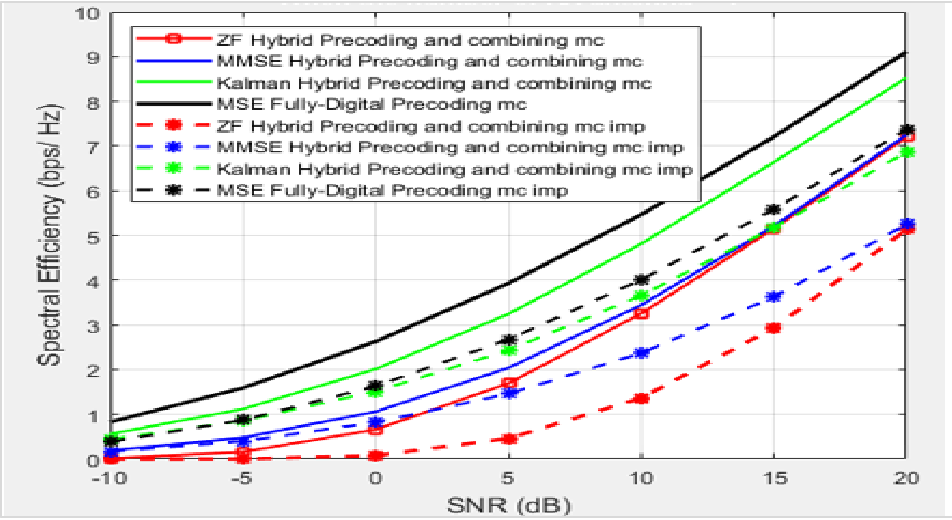

Figure 6 SE for multi-cell based beam-forming schemes under non-ideal conditions (4 antennas per user)

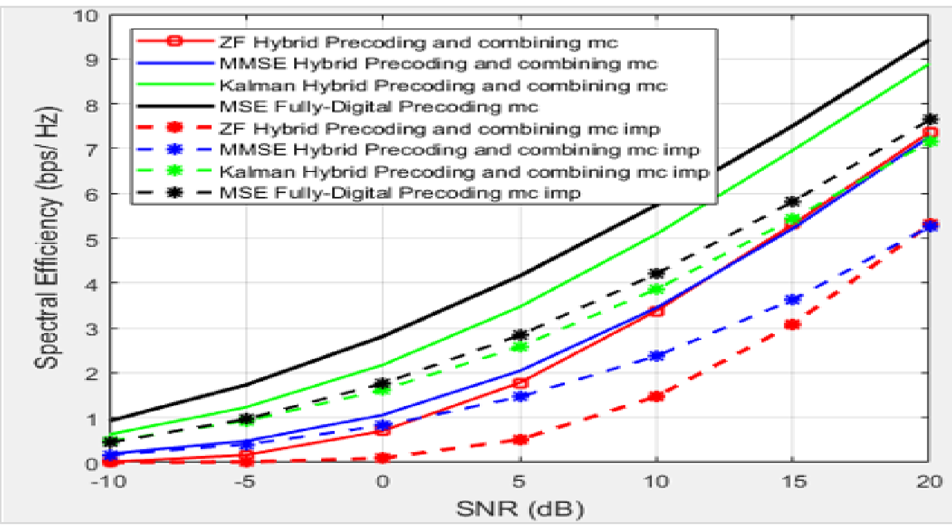

Figure 7 SE for multi-cell based beam-forming under non-ideal conditions (8 antennas per user)

Simulation results in *Figure 8* and *Figure 9* show that the SE of all impairment incorporated beam-formers is affected more when the number of paths increases from 10 in *Figure 8* to 16 in *Figure 9*. This is due to the fact that as the number of channel paths increases, most interference will be created, which in turn decreases the SE. From *Figure 8*, the SE of fully digital, Kalman, MMSE, and zero forcing precoders at an SNR of 20 dB and a number of channel paths of 10 under non-ideal conditions is 7.7 bps/Hz, 7.6 bps/Hz, 5.8 bps/Hz, and 4.6 bps/Hz, respectively. From *Figure 9*, the SE of fully digital, Kalman, MMSE, and zero forcing precoders at an SNR of 20 dB and a number of channel paths of 16 under non-ideal conditions is 7.6 bps/Hz, 7 bps/Hz, 6.4 bps/Hz, and 4 bps/Hz, respectively. There is 0.1 bps/Hz, 0.6 bps/Hz, and 0.6 bps/Hz SE degradation for fully digital MSE, Kalman, and zero-forcing precoders at an SNR of 20 dB when the number of paths is 16, and compared the SE of beam-formers with 10 paths under a multi-cell system.

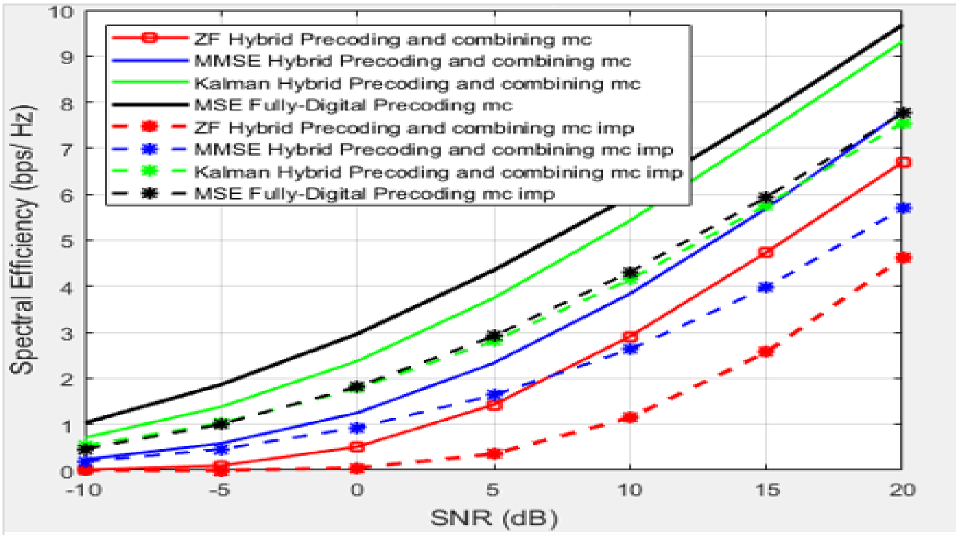

Figure 8 SE for multi-cell based beam-forming under non-ideal conditions (the number of paths = 10)

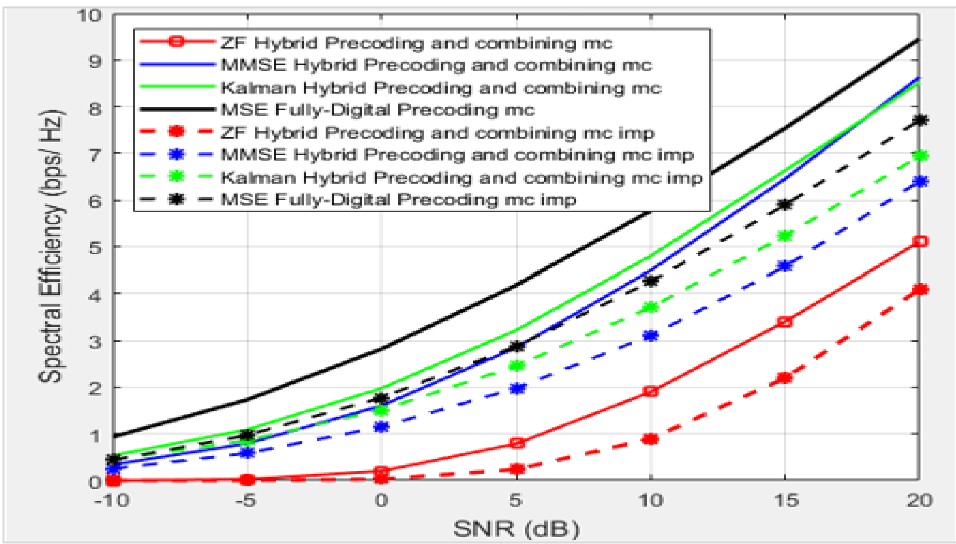

Figure 9 SE for multi-cell-based beam-forming under non-ideal conditions (the number of paths = 16)

5.Discussion

The influence of hardware impairment and incomplete CSI on Kalman-based beam-formers and current linear beam-formers is examined in this work. The number of transmitting antennas, receiving antennas, the number of users, and the number of paths taken into consideration for impairment incorporated Kalman-based multi-cell environment to compare its SE with other linear beam-formers' SE, including fully digital MSE, Kalman, MMSE, and zero forcing. The evaluation was conducted using MATLAB software. There is an improvement in the SE of beam-forming schemes due to an increment in transmitting antennas. Increasing the antenna that is employed at the BS in massive MIMO improves multiplexing gain, which in turn increases SE. From *Figure 2* and *Figure 3*, it can be seen that increasing the antenna number from 128 to 256 improves the SE of all beam formers under non-ideal conditions. There is a degradation in the SE of beam-forming schemes due to the increase in the number of users. Increasing the number of users in massive MIMO degrades SE due to the higher probability of interference associated with increasing users. From *Figure 4* and *Figure 5*, it can be seen that increasing the users' number from 8 to 16 degrades the SE of all beam formers under non-ideal conditions. There is an improvement in the SE of beam-forming schemes due to an increment in receiving antennas. Increasing the antenna that is employed by the user in massive

MIMO improves multiplexing gain, which in turn increases SE. From *Figure 6* and *Figure 7*, it can be seen that increasing the antenna number per user from 4 to 8 improves the SE of all beam formers under non-ideal conditions. There is a degradation in the SE of beam-forming schemes due to the increase in the number of paths. Increasing the number of paths in massive MIMO degrades SE. This is due to the fact that as the number of channel paths increases, most interference will be created, which in turn decreases the SE. From *Figure 8* and *Figure 9*, it can be seen that the increasing number of paths from 10 to 16 degrades the SE of all beam formers under non-ideal conditions. This work will be extended by utilizing the Kalman hierarchical precoder as a precoding scheme for impairment incorporated beam-formers for multi-cell environments.

The limitation of this study is that linear beam-formers including Kalman, MMSE, fully digital MSE and zero forcing computational complexity increases for impairment incorporated multi-cell system even though the SE improved as compared to the single-cell system under the same considerations.

Impairment incorporated Kalman-based hybrid algorithm for multi-cell environment is shown in *Appendix I.* A complete list of abbreviations is shown in *Appendix II.*

6.Conclusion and future work

Beam-forming, along with millimeter wave and massive MIMO is a crucial component to realize 5G communication. Beam-forming for 5G communication under single-cell scenario and ideal assumptions has been intensively investigated. However, beam-forming techniques for multi-cell and non-ideal conditions yet not adequately investigated. This research developed mathematical formulation for multi-cell multi-user scenario based linear beam-formers under imperfect CSI and hardware impairment. It compares linear beam-formers under perfect hardware and CSI with that of imperfect CSI and transceiver hardware impairments. The simulation results from MATLAB software reveal that SE of all beam-formers affected less by increasing transmitting antennas for multi-cell multi-user scenario under non-ideal conditions; SE of all beam-formers for multi-cell multi-user scenario under non-ideal conditions decrease by increasing the number of users; SE of all beam-formers for multi-cell multi-user scenario under non-ideal conditions by increasing receiver antennas; and SE of all beam-formers for multi-cell multi-user scenario under non-ideal conditions decrease by increasing the number of channel paths. The approach will be further developed in the future by including a Kalman hierarchical precoder in a multi-cell, multi-user scenario.

Acknowledgment

None.

Conflicts of interest

The authors have no conflicts of interest to declare.

Author's contribution statement

Tadele A. Abose: Identified the gaps from different literatures, formulate the problem, carried out mathematical model, simulation using matlab software, and prepared the paper. **Thomas O.Olwal and Murad R. Hassen:** Revised the paper, write and proofread it.

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Appendix I

Algorithm 1: Impairment incorporated Kalman-based hybrid algorithm for multi-cell environment

1. Input: F , RF codebooks at BS , W MS RF codebooks
2. Output: $F_{BBk,l}, F_{RFk,l}, W_{BBk,l}, W_{RFk,l}\ \forall_1 = 1, ..., k$
3. Step1: Analog design for each user k in cell L single user
$F_{RFk,l}$ and $W_{RFk,l}$, $\forall_l , \forall_k$ under imperfect CSI and hardware
4. BS and MS_k selects $v_k g_k \forall_k$ so that

5. $g_k, v_k \ \arg\max \ \left\| g_k^H (H_k - \Delta_k) v_k \right\|$

$\forall g_m \in W, \forall v_m \in F$

6. BS sets $F_{RFk,l} = [v_1, ..., v_k]$ and MS_k sets $W_{RFk,l} = g_k \forall_k$

7. Step2: Digital design: multi-cell multi-user precoding and combining under imperfect CSI

8. MS_K channel $\hat{h}_{k,l,i}^H = W_{RFk,l} W_{BBk,l} (H_{k,l,i} - \Delta_k) F_{RFk,l}$ and quantizes H_k using codebook $\mathfrak{H} \forall_k$

9. MS_k Calculates $\hat{h}_k$, $\forall_k$ and sends to BS

10. BS receives $h_{k,l,i} \forall_k$ where

$$h_k = \arg\max \left\| (h_{k,l,i}^H - \Delta_k)(h_{k,l,i} - \Delta_k) \right\| \Delta_k$$

$$\mathrm{h_m} \in H$$

11. BS sets $\hat{H}_D \hat{H}_{ek,l} = \hat{H}[h_{1,l,i} + \Delta_k ... h_{k,l,i} + \Delta_k]^H$

12. For $n \leq N$ do

13. $$e_{k,l}(n) = \frac{I - H_D F_{BB_{k,l}}(n/n-1)}{\left\| I - \hat{H}_D F_{BB_{k,l}}(n/n-1) \right\|_F^2}$$

14. $$F_{BBk,l}(n/n) = F_{BBk,l}(n/n-1) + k(n) \in (n)$$

15. $$e_{k,l}(n) = \frac{I - H_D W_{BB_{k,l}}(n/n-1)}{\left\| I - \hat{H}_D W_{BB_{k,l}}(n/n-1) \right\|_F^2}$$

16. $$W_{BBk,l}(n/n) = W_{BBk,l}(n/n-1) + k(n) \in (n)$$

17. $$k(n) = R(n/n-1)\hat{H}_D^H [\hat{H}_D R(n/n-1)\hat{H}_D^H + Q_n]^{-1}$$

18. $$R(n/n) = [I - k(n)\hat{H}_D R(n/n-1)]$$

19. End

20. Normalize $F_{BBk,l} = \sqrt{P_{k,l}} \dfrac{F_{BBk,l}}{\left\| F_{RFk,l} F_{BBk,l} \right\|_F}$

and

21. $$W_{BBk,l} = \sqrt{P_{k,l}} \frac{W_{BBk,l}}{\left\| W_{RFk,l} W_{BBk,l} \right\|_F}$$

22. Return $W_{BBk,l}, F_{RFk,l}$

Appendix II

S. No.	Abbreviation	Description
1	AoAs	Angles of Arrival
2	AoDs	Angles of Departure
3	ATN	Amplified Thermal Noise
4	BS	Base Station
5	CSI	Channel State Information
6	FD	Full Duplex
7	FDD	Frequency Division Duplex
8	5G	Fifth Generation
9	I-CSI	Imperfect Channel State Information
10	I-SIC	Imperfect Successive Interference Cancellation
11	MIMO	Multiple Input Multiple Output
12	MISO	Multiple Input Single output
13	MRT	Maximum Ratio Transmission
14	MS	Mobile Station
15	MU-MIMO	Multi User Multiple Input Multiple Output
16	QPSs	Quantized Phase Shifters
17	RATHIs	Residual Additive Transceiver Hardware Impairments
18	RF	Radio Frequency
19	RS	Rate Splitting
20	SE	Spectral Efficiency
21	SIC	Successive Interference Cancellation
22	SLP	Symbol Level Precoding
23	SNR	Signal to Noise Ratio
24	TP	Transmission Point
25	UPA	Uniform Phased Array
26	wMMSE	Weighted Minimum-Mean-Square-Error