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Research Article

Forward and inverse kinematics of a 6-DOF robotic manipulator with a prismatic joint using MATLAB robotics toolbox

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Abstract
Robotic manipulators play a crucial role in automating industrial operations, with increasing demand in the manufacturing industry. Investigating the movement of a manipulator with a substantial number of degrees of freedom (DOF) and finding an analytical resolution to the inverse kinematics is paramount in robot modeling. This study focuses on the kinematic modeling and analysis of a 6-DOF robotic manipulator. It aims to validate the accuracy of forward and inverse kinematics calculations using the Denavit-Hartenberg (D-H) parameterization method and MATLAB GUIDE, ensuring precise motion control and path planning for high-precision applications. The 6-DOF robotic manipulator was constructed using SolidWorks, featuring five revolute joints and one prismatic joint. The D-H parameters were established for the manipulator, and kinematic equations were derived. MATLAB GUIDE was employed to perform forward and inverse kinematics calculations, and the results were validated by comparing expected and obtained values. The forward kinematics results demonstrated minimal discrepancies between expected and obtained end-effector positions, with errors ranging from 0.01 to 0.02 units. Inverse kinematics calculations also showed minor deviations in joint angles, generally within 0.01 degrees, indicating a precise match between desired and computed values. These negligible errors confirm the reliability of the D-H parameter assignment and the kinematic equations used. This study successfully simplifies the complex calculations of forward and inverse kinematics for a six-DOF robotic manipulator, providing a robust foundation for precise motion control and path planning. The findings also validate the D-H parameterization method and highlight the practical importance of accurate kinematic modeling in high-precision applications.

Keywords
Robot manipulator, Denavit-Hartenberg (D-H), Forward kinematics, Inverse kinematics, Degree of freedom.

1.Introduction
Robotic manipulators have become pivotal in automating industrial processes, significantly contributing to various sectors, including manufacturing, automotive, aerospace, and healthcare. These mechanical devices, designed to replicate the actions of a human arm, perform tasks through articulated movements with high precision, speed, and efficiency [1, 2]. The evolution from simple single-degrees of freedom (DOF) systems to sophisticated multi-DOF systems has been largely driven by the increasing demand for automation and the need for advanced manufacturing processes. As industries strive for higher productivity and quality, the role of robotic manipulators continues to expand, emphasizing the importance of their development and optimization [2, 3].

Kinematic modeling is a cornerstone of robotics, providing the mathematical framework necessary to design, control, and simulate robotic systems. It focuses on the geometric relationships and movement of robot links and joints, disregarding the forces that cause this movement. This modeling is crucial for understanding, controlling, and simulating the behavior of manipulators [4]. There are two main types of kinematic modeling: forward kinematics and inverse kinematics. Forward kinematics involves determining the position and orientation of the robot's end-effector based on given joint parameters, which is essential for defining the manipulator's reachable workspace. In contrast, inverse kinematics involves calculating the joint parameters required to achieve a specific end-effector position and orientation, presenting a more complex than the direct kinematics problem, especially as the DOF increases, and due to

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the potential for multiple solutions or the absence of a solution [5, 6].

The joints in robotic manipulators are typically classified into two types: revolute joints and prismatic joints. Revolute joints, which allow rotational movement around a single axis, are commonly used in robotic arms, providing versatility and dexterity. Prismatic joints, allowing linear movement along a single axis, are less common but essential in applications requiring linear motion. Combining these joints in a manipulator enhances its ability to perform various tasks with varying complexity and precision [7–9].

Multi-DOF manipulators, especially those with six or more DOF, offer significant advantages in flexibility and reachability. These manipulators can execute complex tasks that require intricate movements and precise positioning. Combining revolute and prismatic joints in a multi-DOF manipulator improves its ability to navigate obstacles and reach targets within constrained environments [10–12]. This versatility is critical in applications requiring high precision and adaptability, such as surgical robotics, where the manipulator must maneuver through tight spaces and adjust its position accurately [13, 14].

The 6-DOF RRRRRP manipulator, which comprises five revolute joints (R) and one prismatic joint (P), represents a unique configuration in robotic manipulators. This configuration poses distinct challenges for kinematic modeling and analysis [15, 16]. The combination of revolute and prismatic joints introduces complexities in determining the manipulator's kinematic behavior, particularly in solving the inverse kinematics problem [17, 18]. While extensive research has been conducted on conventional robotic architectures, there is a lack of comprehensive analysis for manipulators with such unique configurations. Previous studies have often focused on simpler manipulator configurations, neglecting the complexities introduced by prismatic joints and lacking thorough validation through simulation and experimental testing [19]. This gap in the literature leads to uncertainties in the accuracy and applicability of the derived models.

Addressing these challenges, this research aims to develop a robust and efficient kinematic model for the 6-DOF RRRRRP manipulator, validated through extensive simulations using MATLAB tools. By advancing the understanding of robot kinematics for unique configurations, this study contributes to the field of robotics by providing detailed kinematic derivations, comprehensive validation, and analysis of computational efficiency. The primary objectives of this research include deriving a precise and comprehensive kinematic model for the 6-DOF RRRRRP manipulator using the D-H method. It involves accurately defining the geometric relationships and movements of the manipulator's links and joints. Additionally, the aim is to solve the inverse kinematics problem by finding solutions for calculating the joint variables needed to achieve a desired end-effector position and orientation. This step is crucial for ensuring the manipulator can accurately perform tasks requiring precise positioning. Finally, the kinematic model is validated through extensive simulations using the MATLAB Robotics Toolbox. These simulations are conducted to verify the model's accuracy and computational efficiency, ensuring its reliability for practical applications.

This study is structured as follows: Section 2 provides an in-depth overview of the existing research on robot kinematics. Section 3 explains the proposed method. The outcomes and analysis of this research are discussed in Section 4 and 5, and the conclusion is provided in Section 6.

2.Literature survey

Related work has been discussed in this section with the criticial analysis. The study conducted by Wen et al. explored into the simulation of the PRRP setup of a selective compliance articulated robot arm (SCARA) robot with a multi-spindle drilling tool (MSDT) utilizing SolidWorks computer-aided design (CAD) software. The dynamic analysis was also carried out using MATLAB/SimMechanics [20]. Robot manipulator analysis was implemented using MATLAB-Simulink software, focusing on the non-linear sliding mode control (SMC) method [21]. A study on the PUMA 560 robot manipulator focuses on analyzing and implementing non-linear SMC methods using MATLAB/Simulink. The study presented an analytical approach to inverse kinematics using D-H parameters and described creating a software tool in VB6 to understand and manipulate these calculations [22].

Furthermore, the study by Fang and Li employed ANFIS to compute the intricate inverse kinematics of a 4-DOF SCARA robot RRRP, aiming for operational simplicity, maintaining error within acceptable limits, and achieving rapid simulation [23]. The study focused on designing a 6 DOF manipulator for an autonomous aerial mobile robot. The research addressed the challenges of manoeuvrability and stability in aerial robotics by developing a manipulator capable of precise control and flexibility. The manipulator design emphasized minimizing weight while maximizing the DOF to ensure smooth and accurate movements in three-dimensional space. The study's innovative approach aimed to enhance the operational capabilities of aerial robots, enabling them to perform complex tasks with high precision and reliability in various environments [24].

In 2014, vb-based forward and inverse kinematics software was created for a 7-DOF robotic manipulator and compared it to manual computations [25]. A comprehensive kinematic analysis of a 6-DOF Delta manipulator was conducted in this research. This research focused on developing forward and inverse kinematic models to enhance the manipulator's precision and efficiency. The study addressed the complexities of the manipulator's parallel kinematic structure, ensuring the end-effector's accurate position and orientation control. By employing advanced mathematical techniques, the researchers maintained high accuracy while simplifying computational processes, facilitating faster and more reliable simulations essential for high-speed industrial applications [26].

In 2015, a study was conducted on a 4-DOF RRRP SCARA robot using MATLAB for simulation and kinematic analysis [27]. The research conducted by Chen et al. [28] presents an improved inverse kinematics algorithm for a general 6-DOF robot manipulator, utilizing screw theory to address the challenges of solving the inverse kinematics problem, especially in the presence of singularities. Youcef et al. [29] developed a 3D simulation and GUI for the Kawasaki FS03N robot. Bahani et al. explored the use of conformal geometric algebra (CGA) for modeling and controlling the poses of robotic manipulators and legs [30].

In 2016, Senthilkumar and Parthiban used MATLAB to do a kinematic analysis on a 6-DOF articulated robot, showing that it could handle light materials in a low-speed mechanical production system [31]. The study by Xiao et al. proposed an approach to achieve better control performance using a force/position control scheme. It presents a dynamic model of a parallel robot with redundant actuation with 5 DOF and then develops a force/position control scheme using the model predictive control (MPC) algorithm. They used a robot state space model and solve the MPC optimization problem by minimizing a cost function that considers the desired trajectory and system constraints [32]. A new strategy for controlling parallel manipulators without measuring speed was presented in a study. The aim is to solve the problems associated with high acceleration and velocity measurements in parallel manipulators. A practical example of the proposed control strategy was presented and applied to a parallel manipulator with six DOF [33]. The issue of dynamic modeling and parameter estimates for a 7-DOF robot manipulator was explored, namely the Hydraulic arm [34].

In 2017, Xiao et al., focused on the dynamic modeling and simulation of a Mitsubishi RM 501 robot using SolidWorks and MATLAB-Simulink software. They developed an algorithm for comparison sorting, which improved the efficiency and accuracy of the simulation process. Their research demonstrated that the dynamic model could effectively predict the robot's behavior under various operating conditions, offering valuable insights for control design and trajectory planning [32]. Zhang et al. explore applying a backpropagation (BP) neural network to solve the inverse kinematics problem of six DOF serial robots. This research addresses a critical challenge in robotics, where finding the joint angles required to achieve a desired end-effector pose can be complex and computationally intensive [33]. Singh et al. conducted a kinematic analysis of a 6-DOF manipulator arm using MATLAB, obtaining acceptable errors by comparing analytical and software results [35].

In 2018, Angel and Viola presented the grey wolf optimization (GWO) algorithm to fine-tune controller parameters and achieve better performance. This research presents a new approach to controlling a delta-type robotic parallel manipulator using a fractional-order proportional-integral-derivative controller (FOPID). This approach was proposed to solve problems related to parallel robotic manipulators' non-linear dynamics and achieve better tracking control [36]. Pedrammehr et al. focused on the dynamic analysis of the Hexarot, a specific type of axis-symmetric parallel manipulator [37]. In 2019, an experimental study was presented using a non-linear observer to control a highly flexible parallel robot. The aim is to improve the tracking accuracy of a parallel robot by incorporating a non-linear observer into the control strategy. The study presented a practical example in which the proposed control strategy is applied to a parallel robot with three DOF and highly flexible connections. They first derive a dynamic model of the system and then develop a control strategy based on the feedback linearization technique and a non-linear observer. The control strategy is designed to improve the tracking accuracy of the system by compensating for the effects of flexible couplings [38]. Yen et al. provided an innovative, robust adaptive controller for industrial robot manipulators (IRMs) in uncertain dynamical situations utilizing radial basis function neural networks (RBFNNs) [39]. Reboucas et al. presented two methods for resolving singularities that may arise during the movement of a robot arm along a specified trajectory [40].

In 2022, a new control strategy for cable-controlled parallel robots using the non-linear MPC approach was presented. This method was proposed to solve parallel cable-guided robots' non-linear and cable dynamics problems and achieve better control performance [41]. Choubey and Ohri proposed a methodology combining linear quadratic regulator (LQR) and proportional-integral-derivative (PID) controllers to control a parallel manipulator better. The work described a control strategy for improving the performance of a parallel manipulator using a LQR and a PID controller [42]. The study by Ali et al. focused on a specific design for a 6-DOF robot manipulator intended for 3D-printed construction [43]. Xiao et al. utilized a three-dimensional model of the picking manipulator to analyze its position kinematics. Tests in a controlled indoor environment showed that the manipulator's actual movements were similar to those predicted by the simulation software. This comparison confirmed that the 3D model and simulations can accurately predict and control the manipulators' picking actions [44].

In 2023, a new control strategy was proposed for a parallel manipulator with three DOF using a LQR and a PID controller [45]. A new strategy for controlling parallel manipulators using a fast non-linear MPC approach was presented. This approach was proposed to solve problems related to the non-linear dynamics of parallel manipulators and achieve better control performance [46]. It examines a novel approach to guaranteeing a robot's avoidance of obstacles during task completion within a specified time frame. This method represents an advancement over current techniques that do not prioritize time constraints. The research uses simulations and real-world tests on a Baxter robot with seven moving parts to show that the new method works well [47]. Li et al. presented a system for controlling a robotic arm that can move in six ways, designed to help collect and study ice in icy places. The research included simulation and experimental environments to test the manipulator's ability to perform ice drilling and sampling tasks [48]. The research uses kinematic modeling and simulation to focus on the structural design rationality of a 6-DOF manipulator. A 3D model of the manipulator is created on the SolidWorks platform, which is a standard tool for designing mechanical systems. The study concluded that the manipulator's design thrives as the reachable space has no cavities, and the trajectory curve is smooth [49].

In 2024, Karupusamy et al. contributed to robotic welding by providing a thorough kinematic model and performance analysis of a 5-DOF robot designed for this specific application. This research highlighted the importance of kinematic modeling and simulation tools for evaluating the suitability of robots for specific tasks, paving the way for developing more efficient and effective robotic welding systems [50]. The work by Feng et al. made a valuable contribution to the field of construction robotics by providing a detailed analysis of a 6-DOF hydraulic robotic arm specifically designed for side pile driving. This research demonstrates the potential of robotics for automating demanding and hazardous tasks in construction, showcasing the importance of comprehensive kinematic modeling and effective trajectory planning for successful implementation [51].

Raghunathan et al. explored into the kinematic analysis of a curved 6-DOF robotic arm. This research is relevant to the broader field of robotics, particularly in designing and developing manipulators with complex geometries, which can offer advantages regarding workspace, dexterity, and obstacle avoidance [52]. Zhao et al. introduced a fixed-time composite anti-disturbance control framework for n-DOF flexible-link manipulator systems with modeling uncertainties and external disturbances [53]. Using the MATLAB toolbox, Darabseh simulated a two-link rigid-flexible manipulator utilizing Lagrange's equations and the finite elements approach (FE model) [54]. Mary et al. presented an innovative solution to SMC control's main issue, chattering. The controlled system dynamic model and uncertainty upper bound must be known [55]. Ahmed et al. introduced model-free finite-time tracking control for non-linear robotic manipulators. The suggested controller uses time delay estimation (TDE) and enhanced terminal SMC [56].

The reviewed research highlighted a kinematic field of robotics focused on improving robotic manipulators' modeling, control, and applications. Researchers are actively developing sophisticated kinematic and dynamic models, exploring advanced strategies like MPC and adaptive control, and addressing critical challenges like singularity handling and obstacle avoidance. Simulation tools like MATLAB and SolidWorks are vital in testing and optimizing robot designs. Beyond traditional manufacturing applications, research is expanding into diverse fields like construction, welding, and ice exploration, demonstrating the growing potential for robots to automate complex tasks. The integration of Artificial intelligence (AI), machine learning, and geometric algebra promises further advancements in robot intelligence, adaptability, and autonomy, shaping the future of this dynamic field.

3.Methodology

3.1Robot kinematics

Robot kinematics is a field of research that focuses on analyzing and describing the motion exhibited by kinematic chains with multiple DOF, constituting the foundational structure of robotic systems. *Figure 1* shows the relationship between the joint angles and the end-effector's position using forward and inverse kinematics. Forward kinematics uses a robotic system's kinematic equations to calculate the end-effector's precise position using specified joint parameters. Inverse kinematics calculates the joint angles needed to reach a specific end-effector position. The robot's workspace is defined by its dimensions and kinematics equations [57]. The manipulator of six-DOF was built utilizing the SolidWorks software, as shown in *Figure 2*. This manipulator has five links $(H, L1, L2, L3, L4)$, five angles $(q1, q2, q3, q4, q5)$ and one prismatic joint $(q6)$.

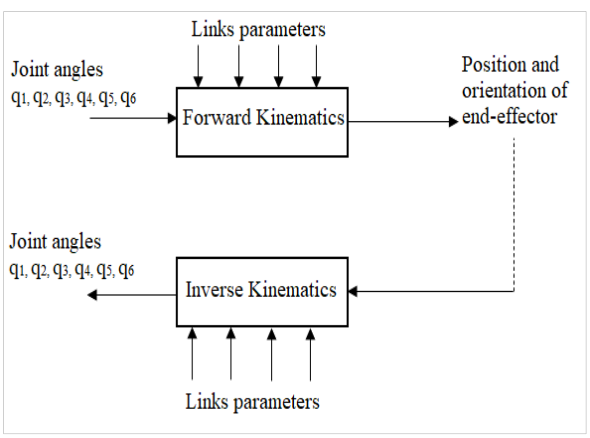

Figure 1 Relationship between joint angles and the position of the end effector

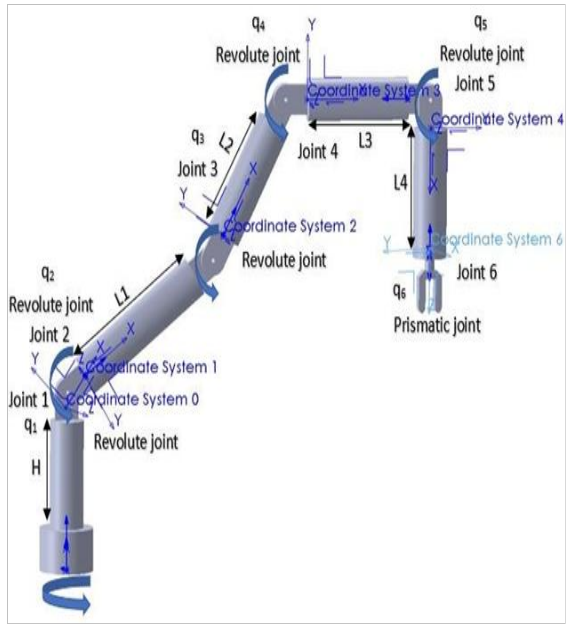

Figure 2 The structure of the 6-DOF manipulator

3.2Work space envelope

The regions that a robot manipulator can reach with its end effector are called its workspace. This is based on the robot's mechanical configuration. *Figure 3* shows the XY plane, which shows the workspace of a robot manipulator. The *Figure 3* shows the robot manipulator's reach based on *Table 1's* limits for each joint. The limits for each joint are shown in *Table 1*.

The notations J_1, J_2, J_3, J_4, J_5 and J_6 represent the different joints in the robot manipulator. The range for revolute joints is given in degrees, and the range for prismatic joints is in meters. *Figure 3* shows the robot manipulator workspace envelope appearing in the XY plane, showing the maximum joint positions based on the joint constraints.

Table 1 Joint motion constraints

Joint No.	Type	Range (deg, meter)
J_1	Revolute	-180 to 180
J_2	Revolute	-60 to 60
J_3	Revolute	-45 to 45
J_4	Revolute	-90 to 90
J_5	Revolute	-45 to 45
J_6	Prismatic	0 to 0.5

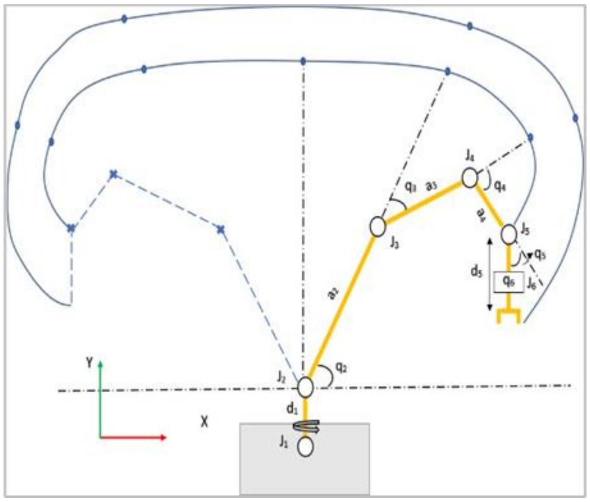

Figure 3 Workspace in the XY plane

3.3Denavit-Hartenberg method

The D-H parameters are the most common frame of reference selection convention in robotics. The four parameters $a_i, \alpha_i, d_i, and\ \theta_i$ are link length, twist angle, joint offset, and joint angle, respectively [58]. In *Table 2*, the manipulator's structural parameters were set out in detail. The representation of every homogeneous transformation Ti is expressed as the result of four fundamental transformations.

Table 2 The denavit- hartenberg parameters

Joint No.	Joint angle θ_i	Joint offset d_i	Link length a_i	Twist angle α_i
-	q1	H	0	0
2	q2	0	0	90º
3	q3	0	L1	0
4	q4	0	L2	0
5	q5	0	L3	0
6	q6	L4	0	90º

The D-H parameters are corresponded to the mounting configuration of links and determined based on the following rule:

- θ_i is the angle from x_{i-1} to xi along z_{i-1};
- d_i is the distance from the intersection of z_{i-1} with x_i to the origin of the (i-1) system of axes;
- a_i is the shortcut between $z_{i-1}$to z_i;
- α_i is the angle from z_{i-1} to z_i along x_i.

3.4Forward kinematic analysis

The framework of the robotic manipulator is developed through the utilization of the D-H parameters and the MATLAB Robotics Toolbox. The transformations between two adjacent joints can be determined by replacing the parameters from the parameters table with the transformation matrix, utilizing Equation 1.

$$A_i = Rot_{z,i}\, Trans_{z,di}\, Trans_{x,ai}\, Rot_{x,\theta i} \quad (1)$$

In forward kinematics analysis, the joint angles determine the end effector's position and orientation by substituting them into the homogenous transformation matrix between joints i and i+1 (Equation 2)[59].

$$A_i^{i+1} = \begin{bmatrix} c_{\theta i} & -s_{\theta i}c_{\alpha i} & s_{\theta i}\, s_{\alpha i} & a_i\, c_{\theta i} \\ s_{\theta i} & c_{\theta i}c_{\alpha i} & -c_{\theta i}s_{\alpha i} & a_i\, s_{\theta i} \\ 0 & s_{\alpha i} & c_{\alpha i} & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (2)$$

by substituting the parameters in *Table 1* to obtain transformation matrices A_1^0 to A_6^5 , can be obtained as shown:

$$A_1^0 = A_1 = \begin{bmatrix} c_1 & -s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & H \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Where the matrix A_1 shows the transformation between frame 1 to frame 0, $c_1 = \cos(q_1)$ and $s_1 = \sin(q_1)$.

$$A_2^1 = \begin{bmatrix} c_2 & 0 & s_2 & 0 \\ s_2 & 0 & -c_2 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3^2 = \begin{bmatrix} c_3 & -s_3 & 0 & c_3.L_1 \\ s_3 & c_3 & 0 & s_3.L_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_4^3 = \begin{bmatrix} c_4 & -s_4 & 0 & c_4.L_2 \\ s_4 & c_4 & 0 & s_4.L_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_5^4 = \begin{bmatrix} c_5 & -s_5 & 0 & c_5.L_3 \\ s_5 & c_5 & 0 & s_5.L_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_6^5 = \begin{bmatrix} c_6 & 0 & -s_6 & 0 \\ s_6 & 0 & c_6 & 0 \\ 0 & 1 & 0 & L_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The total transformation A_6^0 between the base frame and the end-effector frame can be found by successive multiplication:

$$A_6^0 = A_1^0.A_2^1.A_3^2.\ A_4^3.A_6^5 = \begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (3)$$

Where $\{p_x, p_y, p_z\}$ represent the position and $\{(n_x, o_x, a_x), (n_y, o_y, a_y), (n_z, o_y, a_z)\}$ represent the orientation of the end-effector. The calculation of the orientation and position of the end-effector can be determined by using joint angles and the D-H parameters of the manipulator, as shown in Equation 3.

The total transformation A_6^0 is shown in Equation 4-Equation 7.

$$A_6^0 = \begin{bmatrix} s_6 s_{base} + c_{2345}\, c_6\, c_{base} & s_{base} c_6 - c_{2345}\, s_6\, c_{base} & c_1\, s_{2345} & P_x \\ c_{2345}\, s_{base}\, c_6 - c_{base}\, s_6 & -c_{base} c_6 - c_{2345} s_6 s_{base} & s_1\, s_{2345} & P_y \\ s_{2345}\, c_6 & -s_{2345}\, s_6 & -c_{2345} & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (4)$$

$$P_x = c_{base} * (L_1 * c_2 + L_2 * c_{23} + L_3 * c_{234} + L_4 * s_{2345}) \quad (5)$$

$$P_y = s_{base} * (L_1 * c_2 + L_2 * c_{23} + L_3 * c_{234} + L_4 * s_{2345}) \quad (6)$$

$$P_z = H + L_2 * s_{23} + L_1 * s_2 + L_3 * s_{234} - L_4 * c_{2345} \quad (7)$$

Where: $s_{56} = \sin(q_5 + q_6), c_{56} = \cos(q_5 + q_6)$, $c_{1234} = \cos(q_1 + q_2 + q_3 + q_4)$, $s_{1234} = \sin(q_1 + q_2 + q_3 + q_4)$.. Making use of some trigonometric equations helps for easy solutions: $c_{12} = c_1 c_2 - s_1 s_2$, $s_{12} = c_1 s_2 + s_1 c_2$, $C(a+b) = c_a c_b + s_a s_b$, $S(a-b) = c_a s_b + s_a c_b$

3.5Inverse kinematic analysis

Solving inverse kinematics is more complex than the method of solving forward kinematics [60]. Inverse kinematics is a technique for figuring out what values for the joints result in the desired position and orientation of the end effector [61]. When working with inverse kinematics, Equation 3 is multiplied by the inverse of the initial transformation matrix. It allows the calculation of the joint angles by comparing the matrix elements in Equation 8.

$$(T_n^0)^{-1} \times T_n^0 = T_2^1 \ldots T_n^{n-1}; \quad \text{where:} \quad (T_n^0)^{-1} = \frac{Adj\,(T_n^0)}{Det\,(T_n^0)} \quad (8)$$

$$A_1^{-1} = \begin{bmatrix} c_1 & -s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & -H \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_2^{-1} = \begin{bmatrix} c_2 & s_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ s_2 & -c_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3^{-1} = \begin{bmatrix} c_3 & s_3 & 0 & -L_1 \\ -s_3 & c_3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_4^{-1} = \begin{bmatrix} c_4 & s_4 & 0 & -L_2 \\ -s_4 & c_4 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_5^{-1} = \begin{bmatrix} c_5 & s_5 & 0 & -L_3 \\ -s_5 & c_5 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_6^{-1} = \begin{bmatrix} c_6 & s_6 & 0 & 0 \\ -s_6 & c_6 & 0 & 0 \\ 0 & 0 & 1 & -L_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Using the algebraic solution technique, the matrix in Equation 8 can be easily solved by Equation 9.

$$A_6^0 = A_1^0 \,.\, A_2^1 \,.\, A_3^2 \,.\, A_4^3 \,.\, A_6^5 \quad (9)$$

Solutions for q_1 and q_5

To determine the value of q_i given specific numerical values for A_6^0, it is necessary to perform multiplication on both sides by A_1^{-1} (Equation 10).

$$A_1^{-1} * \begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} = A_2^1 * A_3^2 * A_4^3 * A_5^4 * A_6^5 \quad (10)$$

The two elements of the matrix in Equation 10 are set equal (right-hand side = left-hand side), and the resulting values are determined by Equation 11 and 12.

$$c_1 p_x + s_1 p_y = L_1 c_3 c_{245} + L_2 c_4 + L_3 c_{45} + L_4 s_{245} \quad (11)$$

$$-s_1 p_x + c_1 p_y = L_1 c_3 s_{245} + L_2 s_4 + L_3 s_{45} - L_4 c_{245} \quad (12)$$

Squaring and adding the two equations, we can solve for c_5 as follows (Equation 13):

$$c_5 = \frac{p_x^2 + p_y^2 - (L_1 c_3)^2 - L_2^2 - L_3^2 - (L_4\, s_{245})^2}{2 * L_2\, L_3} \quad (13)$$

The s_5 can be solved as follows (Equation 14):

$$s_5 = \mp\sqrt{1 - c_5^2} = \mp\sqrt{1 - \frac{(p_x^2 + p_y^2 - (L_1 c_3)^2 - L_2^2 - L_3^2 - L_4\, s_{245})^2}{(2 * L_2\, L_3)^2}} \quad (14)$$

Square and add Equations 13 and 14 to obtain Equation 15.

$$q_5 = \mathrm{Atan2}(s_5, c_5) \quad (15)$$

Rearrange Equations 11 and 12 to solve for c_1 , s_1 (Equation 16-18):

$$c_1 = \frac{L_1 c_3 c_{245} + L_2 c_4 + L_3 c_{45} + L_4 s_{245} - s_1 p_y}{p_y} \quad (16)$$

$$s_1 = -\frac{L_1 c_3 s_{245} + L_2 s_4 + L_3 s_{45} - L_4 c_{245} - s_1 p_y}{p_x} \quad (17)$$

$$q_1 = \mathrm{Atan2}(s_1, c_1) \quad (18)$$

Solutions for q_3 and q_2

After solving for q_1 and q_5, using the forward kinematics equations simplifies solving for q_3 and q_2

easy. Equation 18 and 19 are obtained based on Equation 4:

$p_z - H = L_1 s_3$

$s_3 = \frac{p_z - H}{L_1}$ (19)

$q_3 = \text{Atan2}\left[\frac{p_z - H}{L_1}, \mp\sqrt{1 - \left(\frac{p_z - H}{L_1}\right)^2}\right]$ (20)

Multiplying each side of Equation 9 with $A_1^{-1} * A_2^{-1}$ (Equation 21):

$$A_1^{-1} * A_2^{-1} * \begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} = A_3^2 * A_4^3 * A_5^4 * A_6^5 \quad (21)$$

Equating elements (3, 4) of the right-hand side matrix and the left-hand side matrix of Equation 20 yields Equation 22:

$s_2\, p_x - c_2\, p_y - H = L_4$

$s_2\, p_x - c_2\, p_y = L_4 + H$

$q_2 = \text{Atan2}(p_x, -p_y) \mp \left[\sqrt{{p_x}^2 + {p_y}^2 - (L_4 + H)^2}, (L_4 + H)\right]$ (22)

Solutions for q_6 and q_4

From Equation 4, the following can be obtained:

$a_x = c_{56}\, s_{1234}$

$a_y = s_{56}\, s_{1234}$

Dividing the two equations:

$\frac{S_{56}}{C_{56}} = \frac{a_y}{a_x} q_{56} = \text{Atan2}(a_y, a_x)$

And then Equation 23 can be obtained:

$q_6 = q_{56} - q_5$ (23)

Now multiply each side of Equation 9 by Equation 24:

$$A_1^{-1} * A_2^{-1} * A_3^{-1} \begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} = A_4^3 * A_5^4 * A_6^5 \quad (24)$$

By comparing the elements at positions 3 and 4 on both sides of Equation 25, Equations 26 and 27 were obtained:

$s_{23}\, p_x - c_{23} p_y - L_1 s_2 - H = L_4$

$s_{23}\, p_x - c_{23} p_y = L_1 s_2 + H + L_4$

$q_{23} = \text{Atan2}(p_x, -p_y) \mp \text{Atan2}\left(\sqrt{p_x^2 + p_y^2 - (L_1 s_2 + H + L_4)^2}, L_1 s_2 + H + L_4\right)$ (25)

$q_3 = q_{23} - q_2$ (26)

From Equation 4, Equation 25 can also be obtained:

$a_z = -c_{1234}$

$C_{1234} = -a_z\, q_{1234} = \text{Atan2}\left(\sqrt{1 - {a_z}^2}, a_z\right)$

$q_4 = q_{1234} - q_2 - q_3$ (27)

Coordinate the systems of the six-DOF manipulator, which are established using the D-H method [62]. To conduct a kinematic analysis, the coordinate frames for each link were determined systematically, beginning at the base and ending at the end effector. The representation of links and frame assignment for the manipulator is shown in *Figure 4.*

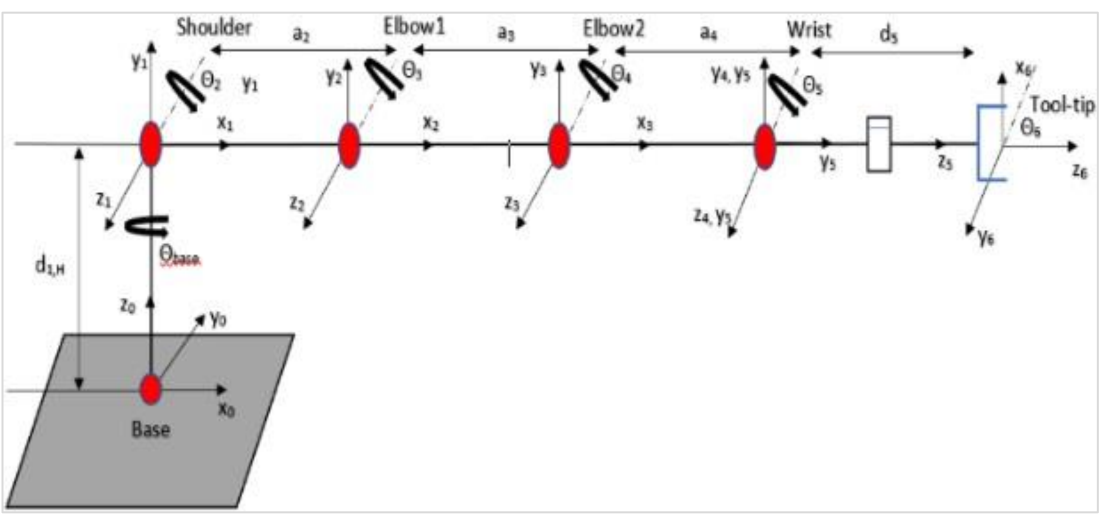

Figure 4 The coordinate systems of the 6-DOF manipulator

3.6Process used to prepare the MATLAB GUIDE for the kinematic analysis

This study used MATLAB and the robotic toolbox developed as software tools to execute kinematic analysis. *Figure 5* displays the flowchart illustrating the flow employed to prepare the MATLAB GUIDE utilized in the kinematic analysis. In this process, the initial step involves the user providing the D-H parameters, consisting of the link length, twist, offset, and joint angle. After that, if the user intends to perform forward kinematics, the joint angles are provided as input.

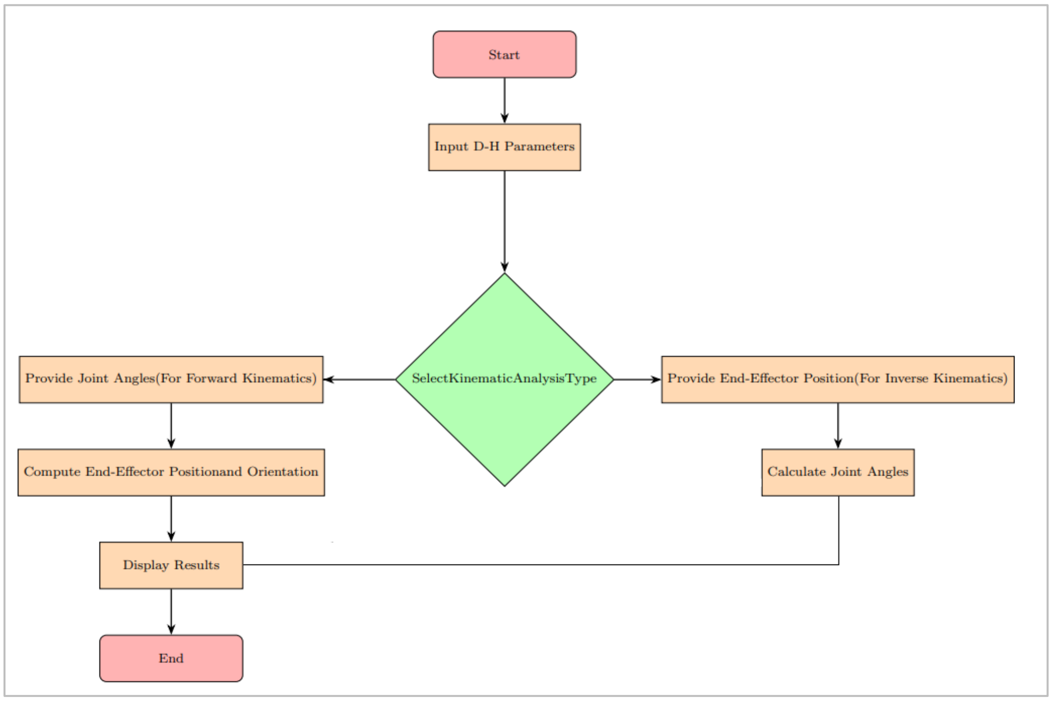

Figure 5 Flowchart of the process used to prepare the MATLAB GUIDE for the kinematic analysis

3.7MATLAB Guide

In this work, a MATLAB GUIDE has been constructed to perform forward kinematic and inverse kinematic analysis. *Figure 6* displays a graphic representation of the MATLAB GUIDE interface. *Figure 7* displays a graphic representation of the RRRRRP manipulator in its initial configuration. *Figure 8* displays a screenshot representing the complete configuration of an RRRRRP manipulator after the computation of forward kinematics.

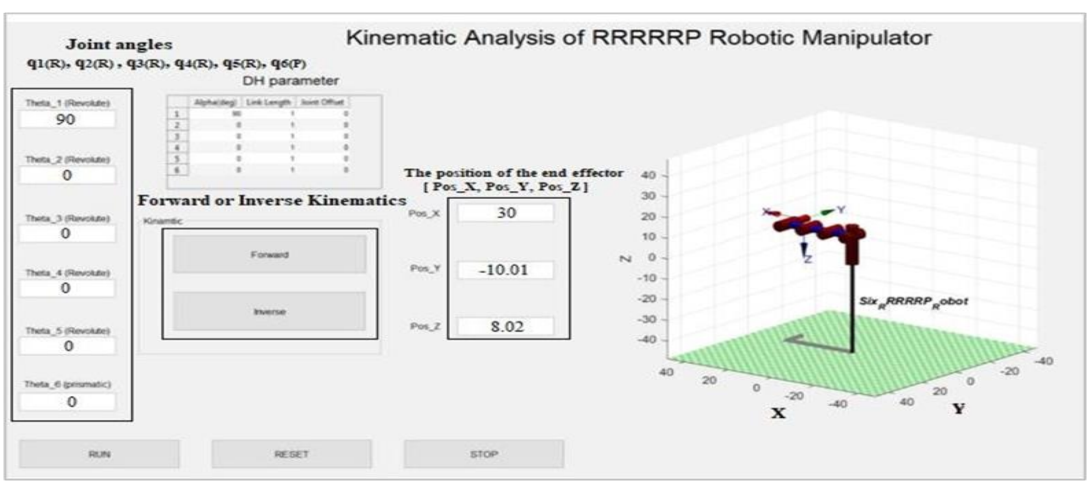

Figure 6 MATLAB guide

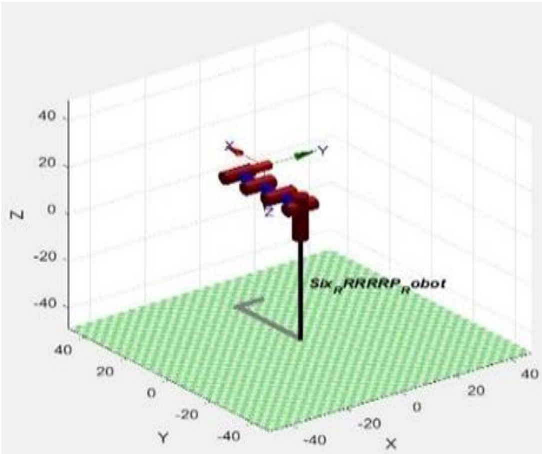

Figure 7 Screenshot of the initial position of the RRRRRP manipulator

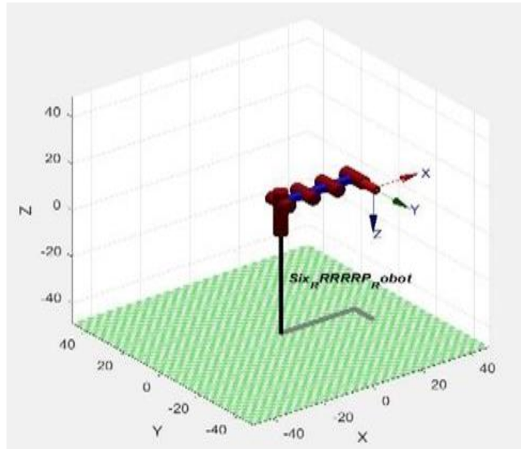

Figure 8 Screenshot of the final position after forward kinematics

4.Results

The forward and inverse kinematics calculations exhibited high accuracy, with minimal errors observed. *Tables 3* and *4* display the expected and obtained findings for forward and inverse kinematics, respectively. The obtained end-effector positions and joint angles closely matched the expected values, confirming the validity of the D-H parameter assignment and the employed kinematic equations. The accurate kinematic modeling presented in this study forms a foundation for precise motion control and path planning, which is crucial for applications requiring accurate positioning and trajectory control.

Table 3 Forward kinematics results

Joints No.	Values		Positions	Values
Θ_1	90		X	30
Θ_2	0	Expected	Y	10
Θ_3	0		Z	8
Θ_4	0		X	30
Θ_5	0	Obtained	Y	10.01
Θ_6	0		Z	8.02

Table 4 Inverse kinematics results

Positions	Values	Expected	Obtained
		Θ_1=90	Θ_1=90
X	10	Θ_2=0	Θ_2=-0.007
		Θ_3=0	Θ_3=0.0094
Y	30	Θ_4=0	Θ_4=-0.0028
		Θ_5=0	Θ_5=-0.0056
Z	8	Θ_6=0	Θ_6=-0.0014

5.Discussion

Robot kinematics are crucial in determining a robot's operational efficiency and performance accuracy. The present study on robot kinematics using forward and inverse kinematic analyses was based on the D-H parameters and implemented in MATLAB. Our findings revealed that forward kinematics accurately calculated the end-effector positions, while inverse kinematics effectively determined joint angles for achieving specific end-effector positions. Integrating D-H parameters with MATLAB facilitated precise kinematic modeling and analysis, demonstrating minimal discrepancies between theoretical predictions and simulation results.

This study examines a six-DOF robotic manipulator's kinematic modeling and analysis, constructed using SolidWorks and analyzed with MATLAB and the MATLAB Robotics Toolbox. The manipulator's structural configuration includes five revolute joints and one prismatic joint, characterized by their respective joint constraints, as shown in *Table 1*. The workspace of the manipulator, illustrated in *Figure 3*, effectively demonstrates the spatial reach achievable within these constraints.

The core methodology revolves around the D-H parameterization, a widely recognized convention for establishing kinematic chains in robotic systems. *Table 2* outlines the D-H parameters, providing a systematic framework for transforming joint parameters into end-effector positions. The forward kinematics approach utilizes these parameters to compute the position and orientation of the end effector based on given joint angles, as detailed in Equation 1. Conversely, inverse kinematics involves determining the joint angles required to achieve a specified end-effector position, a process significantly more complex due to its non-linear nature.

The high accuracy observed in both forward and inverse kinematics underscores the robustness of the kinematic model used in the study. The forward and inverse kinematics calculations were validated using MATLAB GUIDE, with the results in *Tables 2* and *3*. The high accuracy of these results, with minimal errors, confirms the reliability of the D-H parameter assignment and the correctness of the employed kinematic equations. The forward kinematics results show a negligible difference between the expected and obtained end-effector positions, as seen in *Table 2*, where discrepancies are within the 0.01 to 0.02 units. Similarly, in inverse kinematics (*Table 3*), the minor deviations in joint angles indicate a precise match between the desired and computed values. These deviations, generally within 0.01 degrees, show that the specific joint angles required to achieve the target end-effector positions were accurately determined. The close agreement between expected and obtained results validates the applicability of D-H parameters and MATLAB tools for kinematic analysis in robotic systems. This suggests that the theoretical frameworks and computational methods are reliable for simulating and predicting robot motion in controlled environments.

Our findings align with previous research in robotic kinematics, where the D-H parameterization has consistently been demonstrated as an effective method for modeling robotic systems, highlighting the robustness of the D-H approach in both academic and practical applications [63, 64]. Furthermore, the minimal errors observed in this study are comparable to those reported in similar works, which also noted high accuracy in forward and inverse kinematics computations using the D-H framework [65]. Similar studies have demonstrated the effectiveness of these methodologies in accurately predicting robot motion and facilitating kinematic analysis [66].

The accurate kinematic modeling presented in this study forms a crucial foundation for precise motion control and path planning. This is particularly important for applications requiring high precision in positioning and trajectory control, such as surgical robots, industrial automation, and aerospace systems [2]. The validated kinematic equations and D-H parameters can be directly utilized in developing control algorithms, enabling real-time adjustments and optimizations to enhance manipulator performance [67]. Similarly, reliable inverse kinematics support motion planning and trajectory generation, critical for path optimization and obstacle avoidance in dynamic environments [68].

The practical implications of observed errors in the current analyses can significantly impact the robot's performance and mechanical components. Firstly, errors in forward kinematics, where the calculated end-effector positions may deviate slightly from theoretical expectations, can affect the precision of tasks performed by the robot. For instance, in applications requiring precise positioning, such as manufacturing processes or surgical robotics, even small errors in positioning can lead to inaccuracies in product assembly or surgical procedures. These inaccuracies may result in rework, increased production time, or compromised surgical outcomes, highlighting the importance of minimizing such errors through accurate kinematic modeling [69, 70]. Secondly, errors in inverse kinematics, which determine the joint angles necessary to achieve a desired end-effector position, can impact the robot's ability to execute planned trajectories accurately. If the computed joint angles have errors, the robot may not reach the intended positions accurately, affecting its overall task performance [71]. Accurate positioning is crucial in medical robotics, particularly robotic surgery, where precision directly impacts patient safety and surgical outcomes. Small errors in the kinematic model could lead to misalignments, necessitating error compensation techniques to maintain the high accuracy required in such sensitive applications [72].

If not addressed, the minor discrepancies observed in the kinematic model could compromise the reliability of operations in high-stakes environments. Moreover, repeated inaccuracies in both forward and inverse kinematics or the cumulative effect of minor inaccuracies in joint movements can contribute to wear and tear on the robot's mechanical components, potentially reducing the lifespan and reliability of the robotic manipulator [73]. Continuous operation with misaligned positions or movements can increase friction, joint stress, and potential mechanical failures over time. This affects the robot's reliability and increases maintenance costs and repair downtime, impacting overall operational efficiency.

These practical implications require ongoing refinement of kinematic models and control algorithms to reduce errors and enhance accuracy in robotic operations. Advances in sensor technology, feedback control systems, and computational algorithms can mitigate these issues by providing real-time adjustments and corrections during robot operation [74]. Additionally, rigorous testing and validation of kinematic models against practical scenarios and environmental factors are crucial to ensure reliable performance in diverse applications [75]. Addressing sources of error such as numerical precision limitations, modeling simplifications, and mechanical imperfections like joint flexibility and backlash is essential for improving overall accuracy and operational efficiency of robotic systems. Thus, the implications of the observed errors extend beyond mere theoretical concerns, impacting the practical performance, efficiency, and longevity of robotic manipulators in real-world applications [76].

5.1Limitations of the study

While theoretical predictions were compared with simulation results, the study lacked extensive validation in real-world scenarios or with physical prototypes. Real-world testing is crucial to validate the accuracy and reliability of kinematic models under diverse operating conditions and external disturbances. Although a MATLAB GUIDE interface was developed for user-friendly kinematic analysis, practical implementation and usability in industrial or clinical settings may require additional considerations such as integration with existing control systems, user training, and adaptation to specific operational requirements. A complete list of abbreviations is listed in *Appendix I.*

6.Conclusions and future work

This study successfully simplifies the complex calculations of forward and inverse kinematics for a 6-DOF robotic manipulator using the D-H parameterization method and MATLAB GUIDE. The results demonstrate high accuracy, with negligible errors between expected and obtained values, confirming the reliability of the kinematic model. The MATLAB GUIDE tool proved highly effective for kinematic analysis, offering a customized interface and detailed control over simulation parameters. It efficiently generated numerous end positions by inputting various joint angles through forward kinematics and accurately calculated various joint angles by inputting different end positions through inverse kinematics. While MATLAB GUIDE excels in these tasks, MATLAB/Simulink is better suited for extensive simulations involving dynamic modeling and complex control systems. Future research efforts should focus on incorporating dynamic modeling using MATLAB/Simulink to explore variables such as inertia, friction, and external forces. This would enhance understanding and improve control techniques in the dynamics of robotic manipulators, further advancing the field of robotic control systems.

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Conflicts of interest

The authors have no conflicts of interest to declare.

Data availability

None.

Author's contribution statement

M. Y. Alwardat: Conceptualize, investigate, design model, data curation, coding, interpreting results, write and review the paper. **H. M. Alwan:** Writing, results investigation and supervision.

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Appendix I

S. No.	Abbreviation	Description
1	AI	Artificial Intelligence
2	BP	Backpropagation
3	CGA	Conformal Geometric Algebra
4	DOF	Degrees of Freedom
5	FOPID	Fractional-Order Proportional-Integral-Derivative
6	GWO	Grey Wolf Optimization
7	IRMs	Industrial Robot Manipulators
8	MPC	Model Predictive Control
9	MSDT	Multi-Spindle Drilling Tool
10	PID	Proportional-Integral-Derivative
11	RBFNNs	Radial Basis Function Neural Networks
12	SMC	Sliding Mode Control
13	TDE	Time Delay Estimation